Getting the Stochastic Term expected with VPTs

File at http://cmsdoc.cern.ch/~ecendcap/
Click on VPTs section
D Cockerill, 6.8.99

Consider light collection only.
Poisson statistics. Distribution with a mean value of N photoelectrons has a variance of N.

Thus :

$$\frac{\sigma}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$ where N is the mean number of photoelectrons.

Suppose X photoelectrons/GeV then

$$N = X \cdot E\,(\text{GeV})$$

Now the important bit :

$$\frac{\sigma}{N} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{X \cdot E(\text{GeV})}}$$

$$= \frac{1}{\sqrt{X}} \cdot \frac{1}{\sqrt{E(\text{GeV})}}$$

and in percentage terms

$$\frac{\sigma}{N}(\%) = \frac{100}{\sqrt{X}} \cdot \frac{1}{\sqrt{E(\text{GeV})}}$$

If a VPT has an excess noise factor of F then the equation becomes :

$$\frac{\sigma}{N}(\%) = \frac{100}{\sqrt{X}} \cdot \sqrt{\frac{F}{E(\text{GeV})}}$$

$$= 100 \cdot \sqrt{\frac{G}{Y}} \cdot \sqrt{\frac{F}{E(\text{GeV})}}$$

where Y is the crystal + VPT yield, in electrons/GeV and G is the VPT gain
giving $X = Y / G$,the number of photoelectrons / GeV

Note X is dependant on the VPT quantum efficiency, effective area and crystal plus wraping light yield.

Measurements required
A charge injection pulse to calibration in electrons
A test beam measurement to calibrate in GeV !
VPT gain, G, measured in the lab
A measure of F - and this is tricky, not only at B=0 but also at B= 4T.
Typical values as measured in the lab seem to lie between 2 and 3.

Example

Crystal + VPT yield 20 electrons per MeV.
Gain of the VPT, G = 8
Then the photostatistics are 2.5 photoelectrons/MeV = 2500 p.e./GeV.

Then :

$$\frac{\sigma}{N}(\%) = \frac{100}{\sqrt{X}} \cdot \sqrt{\frac{F}{E(\mathrm{GeV})}} = \frac{100}{\sqrt{2500}} \cdot \sqrt{\frac{F}{E(\mathrm{GeV})}}$$

$$= \frac{100}{50} \cdot \sqrt{\frac{F}{E(\mathrm{GeV})}}$$

$$= 2\ \% \cdot \sqrt{\frac{F}{E(\mathrm{GeV})}}$$

If F = 1, the stochastic term, a, = 2 %
If F = 2, a = 2.8 %
If F = 3, a = 3.46
If F = 4, a = 4.0

Other contributions to the final experimental stochastic term

On a 3x3 EE matrix, beam in centre 4x4 mm2, 1.8%/sqrt(E) to be added in quadrature to the above.