The electroweak chiral Lagrangian reanalyzed

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Contents

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1. Introduction

- Gauge symmetry: important concept in particle physics
- Recently we presented a manifestly gauge-invariant approach to the bosonic sector of the Standard Model
  $\rightarrow$ gauge symmetry preserved throughout the calculation

  Nyffeler, Schenk, hep-ph/9812437
- Possible applications:
  - Inclusion of finite width effects of unstable particles, e.g. in W-pair production
  - Effective Action Γ in gauge theories $\leftrightarrow$ Electroweak phase transition
  - Treatment of electromagnetic corrections in Chiral Perturbation Theory
  - Effective Lagrangian approach to the symmetry breaking sector of the SM (this talk + work in preparation)

2. Effective field theory in the EW sector

EW symmetry breaking sector: unknown mechanism
(SM, SUSY, Technicolor, ...)
General idea: use $\mathcal{L}_{eff}$ to parametrize full theory at low energies in model independent way

$$\mathcal{L}_{eff} = \sum_i l_i \mathcal{O}_i$$

$\mathcal{O}_i$: operators built from light particles in spectrum; $SU(2)_L \times U(1)_Y$ gauge-invariant

l_i: low energy constants (LEC's); encode full theory; comparison $l_i^{exp} \leftrightarrow l_i^{th}$

1. Decoupling case
   Light Higgs $\Rightarrow \mathcal{L}_{eff} = \mathcal{L}_{SM} + \delta\mathcal{L}$
   $\mathcal{L}_{SM}$ renormalizable $\Rightarrow \delta\mathcal{L}$ composed of higher dimensional operators suppressed by $1/\Lambda, 1/\Lambda^2, \ldots$ (Λ: scale of new physics)
   $SU(2)_L \times U(1)_Y$ symmetry linearly realized
   Buchmüller, Wyler (1986)

2. Non-decoupling case
   Heavy Higgs; no (fundamental) Higgs: strongly interacting EW symmetry breaking sector
   $\mathcal{L}_{eff}$ = EW chiral Lagrangian
   Gauge symmetry non-linearly realized

The electroweak chiral Lagrangian

- Strongly interacting symmetry breaking sector
$\rightarrow$ Useful or even necessary to employ effective field theory description in analogy to Chiral Perturbation Theory for QCD at low energies

  Appelquist, Bernard (1980); Longhitano (1981); Appelquist, Wu (1993, 1995)

- Here: gauge fields only, no matter fields

- Problem in gauge theories: usually one is working with gauge-dependent Green's functions of gauge fields:

$$\mathcal{L}_{eff} = \mathcal{L}_{eff}^{inv} + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

  $\mathcal{L}_{eff}$ should include all BRS-invariant terms

- Issue of gauge invariance important in following cases:
  - Number of independent LEC's in $\mathcal{L}_{eff}$ $\leftrightarrow$ removing terms by using the equations of motion (EoM)
  - Evaluate LEC's for a given theory $\leftrightarrow$ matching full and effective field theory at low energies (similarly: integrating out heavy particles)

- Proposition: Study Green's functions of gauge-invariant fields

  Tool: Generating Functional $\rightarrow$ well defined framework

  cf. Gasser, Leutwyler (1984, 1985) for Chiral Perturbation Theory

The electroweak chiral Lagrangian (matrix notation)

Expansion in powers of momentum:

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots, \quad \mathcal{L}_n = \mathcal{O}(p^n)$$

Order p^2:

$$\begin{aligned} \mathcal{L}_2 &= \frac{1}{4}v^2 \mathrm{tr}(\hat{D}_\mu \hat{U}^\dagger \hat{D}_\mu \hat{U}) - (\bar{\rho}-1)\frac{v^2}{8}\left[\mathrm{tr}(\hat{T}\hat{V}_\mu)\right]^2 \\ &+ \frac{1}{2g^2}\mathrm{tr}(\hat{W}_{\mu\nu}\hat{W}_{\mu\nu}) + \frac{1}{2g'^2}\mathrm{tr}(\hat{B}_{\mu\nu}\hat{B}_{\mu\nu}) \end{aligned}$$

$SU(2)$-matrix notation:

$$\begin{aligned} \hat{U} &= \exp\left(i\frac{\tau^a \pi^a}{v}\right) \in SU(2) \\ \hat{W} &= W_\mu^a \frac{\tau^a}{2} \\ \hat{B} &= B_\mu \frac{\tau^3}{2} \\ \hat{D}_\mu \hat{U} &= \partial_\mu \hat{U} - i\hat{W}_\mu \hat{U} + i\hat{U}\hat{B}_\mu \\ \hat{W}_{\mu\nu} &= \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\nu - i[\hat{W}_\mu, \hat{W}_\nu] \end{aligned}$$

Building blocks for constructing $\mathcal{L}_{eff}$:

$$\begin{aligned} \hat{T} &= \hat{U}\tau^3\hat{U}^\dagger \\ \hat{V}_\mu &= (\hat{D}_\mu \hat{U})\hat{U}^\dagger \\ \hat{\mathcal{D}}_\mu \hat{V}_\nu &= \partial_\mu \hat{V}_\nu - i[\hat{W}_\mu, \hat{V}_\nu] \end{aligned}$$

Order p^4

CP-even terms (Appelquist, Wu (1993): $L_0, \ldots, L_{14}$)

$$
\begin{aligned}
\mathcal{L}_4 &= \sum_{i=0}^{17} a_i L_i \\
L_0 &= \frac{v^2}{4} g'^2 \left[\mathrm{tr}(\hat{T}\hat{V}_\mu)\right]^2 \\
L_1 &= -\frac{1}{2} B_{\mu\nu} \mathrm{tr}(\hat{T}\hat{W}_{\mu\nu}) \\
L_2 &= i\frac{1}{2} B_{\mu\nu} \mathrm{tr}(\hat{T}[\hat{V}_\mu, \hat{V}_\nu]) \\
L_3 &= -i\mathrm{tr}(\hat{W}_{\mu\nu}[\hat{V}_\mu, \hat{V}_\nu]) \\
L_4 &= -\left[\mathrm{tr}(\hat{V}_\mu\hat{V}_\nu)\right]^2 \\
L_5 &= -\left[\mathrm{tr}(\hat{V}_\mu\hat{V}_\mu)\right]^2 \\
L_6 &= -\mathrm{tr}(\hat{V}_\mu\hat{V}_\nu)\mathrm{tr}(\hat{T}\hat{V}_\mu)\mathrm{tr}(\hat{T}\hat{V}_\nu) \\
L_7 &= -\mathrm{tr}(\hat{V}_\mu\hat{V}_\mu)\left[\mathrm{tr}(\hat{T}\hat{V}_\nu)\right]^2 \\
L_8 &= \frac{1}{4}\left[\mathrm{tr}(\hat{T}\hat{W}_{\mu\nu})\right]^2 \\
L_9 &= -i\frac{1}{2}\mathrm{tr}(\hat{T}\hat{W}_{\mu\nu})\mathrm{tr}(\hat{T}[\hat{V}_\mu, \hat{V}_\nu]) \\
L_{10} &= -\left[\mathrm{tr}(\hat{T}\hat{V}_\mu)\mathrm{tr}(\hat{T}\hat{V}_\mu)\right]^2 \\
L_{11} &= -\mathrm{tr}((\hat{\mathcal{D}}_\mu\hat{V}_\mu)^2) \\
L_{12} &= -\mathrm{tr}(\hat{T}\hat{\mathcal{D}}_\mu\hat{\mathcal{D}}_\nu\hat{V}_\nu)\mathrm{tr}(\hat{T}\hat{V}_\mu) \\
L_{13} &= -\frac{1}{2}\left[\mathrm{tr}(\hat{T}\hat{\mathcal{D}}_\mu\hat{V}_\nu)\right]^2 \\
L_{14} &= \epsilon_{\mu\nu\rho\sigma}\mathrm{tr}(\hat{W}_{\mu\nu}\hat{V}_\rho)\mathrm{tr}(\hat{T}\hat{V}_\sigma) \\
L_{15} &= -M_W^2\mathrm{tr}(\hat{V}_\mu\hat{V}_\mu) \\
L_{16} &= \mathrm{tr}(\hat{W}_{\mu\nu}\hat{W}_{\mu\nu}) \\
L_{17} &= \mathrm{tr}(\hat{B}_{\mu\nu}\hat{B}_{\mu\nu})
\end{aligned}
$$

3. A manifestly gauge-invariant approach

Consider bosonic sector of SM without Higgs

Light fields: 3 massive gauge bosons $W_\mu^\pm, Z_\mu$
1 massless photon A_μ

Introduce $SU(2)_L$ gauge-invariant composite fields:

$$\begin{aligned} \mathcal{W}_\mu^+ &= i\,\widetilde{U}^\dagger D_\mu U \quad , \quad \mathcal{W}_\mu^- = i\,U^\dagger D_\mu \widetilde{U} \\ \mathcal{Z}_\mu &= -2i\,U^\dagger D_\mu U \quad , \quad \mathcal{A}_\mu = B_\mu + s^2 \mathcal{Z}_\mu \end{aligned}$$

using the $SU(2)_L$ doublet:

$$U^\dagger U = 1, \qquad \widetilde{U} = i\tau_2 U$$

$$D_\mu U = \left(\partial_\mu - i\frac{\tau^a}{2} W_\mu^a - i\frac{1}{2} B_\mu\right) U$$

Jegerlehner, Fleischer (1985); Chanowitz, Golden, Georgi (1987)

Fields transform under $U(1)_Y$ like in Abelian model:

$$\mathcal{W}_\mu^\pm \to e^{\mp i\alpha(x)} \mathcal{W}_\mu^\pm ; \mathcal{Z}_\mu \to \mathcal{Z}_\mu ; \mathcal{A}_\mu \to \mathcal{A}_\mu - \partial_\mu \alpha$$

Define $SU(2)_L \times U(1)_Y$ gauge-invariant fields:

$$\varphi^\mp \mathcal{W}_\mu^\pm, \; \mathcal{Z}_\mu \; , \mathrm{PT}_{\mu\nu} \mathcal{A}_\nu$$

$$\begin{aligned} \varphi^\pm(x) &= \exp\left(\mp i \int d^d y\, \mathcal{G}_0(x-y)\, \partial_\rho B_\rho(y)\right) \\ \mathcal{G}_0(x-y) &= \langle x | \tfrac{1}{-\Box} | y \rangle \\ \mathrm{PT}_{\mu\nu} &= \delta_{\mu\nu} - \tfrac{\partial_\mu \partial_\nu}{\Box} \end{aligned}$$

The effective Lagrangian

Expansion in powers of momentum:

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots, \quad \mathcal{L}_n = \mathcal{O}(p^n)$$

At lowest order: gauged non-linear sigma model

$$\begin{aligned}\mathcal{L}_2 \; = \; & \frac{v^2}{2}\left(\mathcal{W}_\mu^+\mathcal{W}_\mu^- + \bar{\rho}\frac{1}{4}\mathcal{Z}_\mu\mathcal{Z}_\mu\right) + \frac{1}{4g^2}\mathcal{W}_{\mu\nu}^a\mathcal{W}_{\mu\nu}^a + \frac{1}{4g'^2}B_{\mu\nu}B_{\mu\nu} \\ & -\frac{1}{2}K_{\mu\nu}B_{\mu\nu} + 2v^2(J_\mu^+\varphi^+\mathcal{W}_\mu^- + J_\mu^-\varphi^-\mathcal{W}_\mu^+) + v^2 J_\mu^{\mathcal{Z}}\mathcal{Z}_\mu \\ & +4\bar{c}_{\mathcal{W}}v^2 J_\mu^+ J_\mu^- + \bar{c}_{\mathcal{Z}}v^2 J_\mu^{\mathcal{Z}} J_\mu^{\mathcal{Z}}\end{aligned}$$

where

$$\mathcal{W}_{\mu\nu}^a = \partial_\mu\mathcal{W}_\nu^a - \partial_\nu\mathcal{W}_\mu^a + \varepsilon^{abc}\mathcal{W}_\mu^b\mathcal{W}_\nu^c$$

Sources $K_{\mu\nu}, J_\mu^\pm, J_\mu^{\mathcal{Z}}$ coupled to gauge-invariant fields

Chiral momentum counting:

$$\begin{aligned} U, \varphi^\pm, v, \bar{\rho}-1, s, c, K_{\mu\nu} \; &= \; \mathcal{O}(p^0) \\ D_\mu, B_\mu, \mathcal{W}_\mu^\pm, \mathcal{Z}_\mu, \mathcal{A}_\mu, M_W, M_Z, g, g', e, J_\mu^a \; &= \; \mathcal{O}(p) \\ \mathcal{W}_{\mu\nu}^a, B_{\mu\nu} \; &= \; \mathcal{O}(p^2)\end{aligned}$$

$g, g', e = \mathcal{O}(p)$ differs from usual dimensional counting
$\bar{\rho} = c^2 M_Z^2/M_W^2$ inverse of usual ρ-parameter

$$M_W^2 = \frac{v^2 e^2}{4s^2}\,, \; M_Z^2 = \bar{\rho}\frac{v^2 e^2}{4s^2c^2}\,, \; c^2 = \frac{g^2}{g^2+g'^2}\,, e^2 = \frac{g^2 g'^2}{g^2+g'^2}$$

Effective Lagrangian at order p^4

Terms without sources (CP-even):

$$
\begin{aligned}
\mathcal{L}_4^0 &= \sum_{i=1}^{18} l_i \mathcal{O}_i \\
\mathcal{O}_1 &= (\mathcal{Z}_\mu \mathcal{Z}_\mu)(\mathcal{Z}_\nu \mathcal{Z}_\nu) \\
\mathcal{O}_2 &= (\mathcal{W}_\mu^+ \mathcal{W}_\mu^-)(\mathcal{W}_\nu^+ \mathcal{W}_\nu^-) \\
\mathcal{O}_3 &= (\mathcal{W}_\mu^+ \mathcal{W}_\nu^-)(\mathcal{W}_\mu^+ \mathcal{W}_\nu^-) \\
\mathcal{O}_4 &= (\mathcal{Z}_\mu \mathcal{Z}_\mu)(\mathcal{W}_\nu^+ \mathcal{W}_\nu^-) \\
\mathcal{O}_5 &= (\mathcal{Z}_\mu \mathcal{Z}_\nu)(\mathcal{W}_\mu^+ \mathcal{W}_\nu^-) \\
\mathcal{O}_6 &= i B_{\mu\nu}(\mathcal{W}_\mu^+ \mathcal{W}_\nu^- - \mathcal{W}_\nu^+ \mathcal{W}_\mu^-) \\
\mathcal{O}_7 &= i \mathcal{Z}_{\mu\nu}(\mathcal{W}_\mu^+ \mathcal{W}_\nu^- - \mathcal{W}_\nu^+ \mathcal{W}_\mu^-) \\
\mathcal{O}_8 &= i \mathcal{Z}_\mu (d_\mu \mathcal{W}_\nu^+ \mathcal{W}_\nu^- - d_\mu \mathcal{W}_\nu^- \mathcal{W}_\nu^+) \\
\mathcal{O}_9 &= i \mathcal{Z}_\nu (d_\mu \mathcal{W}_\mu^+ \mathcal{W}_\nu^- - d_\mu \mathcal{W}_\mu^- \mathcal{W}_\nu^+) \\
\mathcal{O}_{10} &= \epsilon_{\mu\nu\rho\sigma} \mathcal{Z}_\sigma (\mathcal{W}_\rho^- \mathcal{W}_{\mu\nu}^+ + \mathcal{W}_\rho^+ \mathcal{W}_{\mu\nu}^-) \\
\mathcal{O}_{11} &= B_{\mu\nu} \mathcal{Z}_{\mu\nu} \\
\mathcal{O}_{12} &= \mathcal{Z}_{\mu\nu} \mathcal{Z}_{\mu\nu} \\
\mathcal{O}_{13} &= (\partial_\mu \mathcal{Z}_\mu)(\partial_\nu \mathcal{Z}_\nu) \\
\mathcal{O}_{14} &= (d_\mu \mathcal{W}_\mu^+)(d_\nu \mathcal{W}_\nu^-) \\
\mathcal{O}_{15} &= M_W^2 \left(\mathcal{W}_\mu^+ \mathcal{W}_\mu^- + \frac{1}{4} \mathcal{Z}_\mu \mathcal{Z}_\mu \right) \\
\mathcal{O}_{16} &= M_Z^2 \mathcal{Z}_\mu \mathcal{Z}_\mu \\
\mathcal{O}_{17} &= \mathcal{W}_{\mu\nu}^a \mathcal{W}_{\mu\nu}^a \\
\mathcal{O}_{18} &= B_{\mu\nu} B_{\mu\nu}
\end{aligned}
$$

- $\mathcal{O}_1, \ldots, \mathcal{O}_{14}$ can be expressed through the operators $L_1, \ldots, L_{14}$ in usual basis; $\mathcal{O}_{16}$ corresponds to term L_0
- $\mathcal{O}_{15}, \ldots, \mathcal{O}_{18}$ proportional to terms in $\mathcal{L}_2$

Source terms:

$$
\begin{aligned}
\mathcal{L}_4^s &= \sum_{j=1}^{7} l_j^s \mathcal{O}_j^s + \text{contact terms} \\
\mathcal{O}_1^s &= \mathcal{W}_{\mu\nu}^+ j_{\mu\nu}^- + \mathcal{W}_{\mu\nu}^- j_{\mu\nu}^+ \\
\mathcal{O}_2^s &= \mathcal{Z}_{\mu\nu} J_{\mu\nu}^{\mathcal{Z}} \\
\mathcal{O}_3^s &= B_{\mu\nu} J_{\mu\nu}^{\mathcal{Z}} \\
\mathcal{O}_4^s &= (d_\mu \mathcal{W}_\mu^+)(d_\nu j_\nu^-) + (d_\mu \mathcal{W}_\mu^-)(d_\nu j_\nu^+) \\
\mathcal{O}_5^s &= (\partial_\mu \mathcal{Z}_\mu)(\partial_\nu J_\nu^{\mathcal{Z}}) \\
\mathcal{O}_6^s &= M_W^2 (\mathcal{W}_\mu^+ j_\mu^- + \mathcal{W}_\mu^- j_\mu^+) \\
\mathcal{O}_7^s &= M_Z^2 \mathcal{Z}_\mu J_\mu^{\mathcal{Z}}
\end{aligned}
$$

with

$$
\begin{aligned}
\mathcal{W}_{\mu\nu}^\pm &= d_\mu \mathcal{W}_\nu^\pm - d_\nu \mathcal{W}_\mu^\pm \ , \ d_\mu \mathcal{W}_\nu^\pm = (\partial_\mu \mp i B_\mu) \mathcal{W}_\nu^\pm \\
\mathcal{Z}_{\mu\nu} &= \partial_\mu \mathcal{Z}_\nu - \partial_\nu \mathcal{Z}_\mu \ , \ j_\mu^\pm = \varphi^\pm J_\mu^\pm \\
j_{\mu\nu}^\pm &= d_\mu j_\nu^\pm - d_\nu j_\mu^\pm \ , \ J_{\mu\nu}^{\mathcal{Z}} = \partial_\mu J_\nu^{\mathcal{Z}} - \partial_\nu J_\mu^{\mathcal{Z}}
\end{aligned}
$$

- The contact terms do not contribute to physical S-matrix elements (on-shell gauge bosons)
- The terms $\mathcal{O}_3^s, \mathcal{O}_6^s, \mathcal{O}_7^s$ are proportional to source terms in $\mathcal{L}_2$

Generating functional up to one loop

- Path integral representation:

$$e^{-W_{eff}[K_{\mu\nu},J_\mu^a]} = \int d\mu[U,W_\mu^a,B_\mu] \; e^{-\int d^dx \, \mathcal{L}_{eff}}$$

- Saddle-point approximation around solution of classical EoM. Formally:

$$W_{eff}[K_{\mu\nu}, J_\mu^a] = S_{cl} + \frac{1}{2}\mathrm{lndet}' D - \frac{1}{2}\mathrm{lndet} P^T P$$

  – Tree level contributions: classical action S_{cl}
  – One-loop contributions: Gaussian integral
  $\rightarrow$ determinant of differential operator D
  Subtleties due to gauge-invariance: zero-modes corresponding to gauge transformations: $DP = P^T D = 0$, $\det' =$ product of non-zero eigenvalues

- Effective field theory:
  Loop expansion $\equiv$ momentum expansion
  – $\mathcal{O}(p^2)$: tree-level contributions from $\mathcal{L}_2$
  – $\mathcal{O}(p^4)$: tree-level contributions from $\mathcal{L}_4$ and one-loop contributions from $\mathcal{L}_2$

Tree level: Equations of motion (EoM)

- From $\mathcal{L}_2$ we obtain EoM for all fields W_μ^a, B_μ, U (3 Goldstone bosons)
- Sources coupled to gauge-invariant fields $\rightarrow$ EoM uniquely determine the physical degrees of freedom
- Can solve EoM for physical degrees of freedom without the need to fix a gauge, e.g.:

$$(\varphi^\mp \mathcal{W}_\mu^\pm)^T = \int d^dy \frac{1}{-\Box + M_W^2}(-4M_W^2 J_\mu^{\pm,T}), \quad V_\mu^T = \mathrm{PT}_{\mu\nu} V_\nu$$
$$(\varphi^\mp \mathcal{W}_\mu^\pm)^L = -4J_\mu^{\pm,L}, \quad V_\mu^L = \frac{\partial_\mu \partial_\nu}{\Box} V_\nu$$

Nyffeler, Schenk, hep-ph/9812437

- Gauge invariance manifest
  - EoM for the Goldstone bosons follow as constraint equations from EoM for gauge fields
  - EoM do not determine Goldstone boson field U, i.e. $SU(2)_L$ gauge degrees of freedom $\rightarrow$ Goldstone bosons do not propagate at tree-level. They propagate, however, at the quantum level within loops. Approach not equivalent to unitary gauge
  - EoM do not determine B_μ^L and phase factor $\varphi^\pm$, i.e. $U(1)_Y$ gauge degree of freedom
- Whole class of solutions in terms of original fields

Equations of motion (explicit form)

EoM for the gauge fields:

$$-d_\mu \mathcal{W}^{\pm}_{\mu\nu} = -M_W^2(\mathcal{W}^{\pm}_\nu + 4j^{\pm}_\nu) \pm i(\mathcal{Z}_{\mu\nu} + B_{\mu\nu})\mathcal{W}^{\pm}_\mu \mp i\mathcal{W}^{\pm}_{\mu\nu}\mathcal{Z}_\mu$$
$$\mp i(\partial_\mu \mathcal{Z}_\mu)\mathcal{W}^{\pm}_\nu \pm i(\partial_\mu \mathcal{Z}_\nu)\mathcal{W}^{\pm}_\mu \pm i\mathcal{Z}_\nu d_\mu \mathcal{W}^{\pm}_\mu \mp i\mathcal{Z}_\mu d_\mu \mathcal{W}^{-}_\nu$$
$$-(\mathcal{Z}_\mu \mathcal{Z}_\mu)\mathcal{W}^{\pm}_\nu + (\mathcal{Z}_\mu \mathcal{Z}_\nu)\mathcal{W}^{\pm}_\mu \pm 2\mathcal{W}^{-}_\mu(\mathcal{W}^{+}_\mu \mathcal{W}^{-}_\nu - \mathcal{W}^{+}_\nu \mathcal{W}^{-}_\mu)$$

$$-\partial_\mu(\mathcal{Z}_{\mu\nu} + B_{\mu\nu}) = -c^2 M_Z^2(\mathcal{Z}_\nu + \tfrac{4}{\bar\rho} J^{\mathcal{Z}}_\nu) - 4\mathcal{Z}_\mu \mathcal{W}^{+}_\rho \mathcal{W}^{-}_\rho$$
$$+2\mathcal{Z}_\rho(\mathcal{W}^{+}_\rho \mathcal{W}^{-}_\mu + \mathcal{W}^{+}_\mu \mathcal{W}^{-}_\rho) + 2i(\mathcal{W}^{+}_{\rho\mu}\mathcal{W}^{-}_\rho - \mathcal{W}^{-}_{\rho\mu}\mathcal{W}^{+}_\rho)$$
$$-2i(d_\rho \mathcal{W}^{+}_\rho \mathcal{W}^{-}_\mu - d_\rho \mathcal{W}^{-}_\rho \mathcal{W}^{+}_\mu - d_\rho \mathcal{W}^{+}_\mu \mathcal{W}^{-}_\rho + d_\rho \mathcal{W}^{-}_\mu \mathcal{W}^{+}_\rho)$$

$$-\partial_\mu B_{\mu\nu} = s^2 M_Z^2 \mathrm{PT}_{\nu\mu}(\mathcal{Z}_\mu + \tfrac{4}{\bar\rho} J^{\mathcal{Z}}_\mu) - \tfrac{e^2}{c^2}\partial_\mu K_{\mu\nu}$$

EoM for the Goldstone bosons (constraints):

$$d_\mu \mathcal{W}^{\pm}_\mu = -4 d_\mu j^{\pm}_\mu \pm i\mathcal{Z}_\mu(\mathcal{W}^{\pm}_\mu + 4j^{\pm}_\mu) \mp i\bar\rho(\mathcal{Z}_\mu + \tfrac{4}{\bar\rho} J^{\mathcal{Z}}_\mu)\mathcal{W}^{\pm}_\mu$$
$$\partial_\mu \mathcal{Z}_\mu = -\tfrac{4}{\bar\rho}\partial_\mu J^{\mathcal{Z}}_\mu + \tfrac{8i}{\bar\rho}(\mathcal{W}^{+}_\mu j^{-}_\mu - \mathcal{W}^{-}_\mu j^{+}_\mu)$$

4. Number of independent parameters in $\mathcal{L}_{eff}$

- $\mathcal{L}_4$ only contributes at tree-level $\rightarrow$ can use EoM from $\mathcal{L}_2$ to find relations between operators $\mathcal{O}_i$, including source terms $\rightarrow$ can reduce number of terms in $\mathcal{L}_4$:
  - Using the constraint equations one can remove:

$$\begin{aligned}
\mathcal{O}_9 &= i\mathcal{Z}_\nu(d_\mu\mathcal{W}_\mu^+\mathcal{W}_\nu^- - d_\mu\mathcal{W}_\mu^-\mathcal{W}_\nu^+) \\
\mathcal{O}_{13} &= (\partial_\mu\mathcal{Z}_\mu)(\partial_\nu\mathcal{Z}_\nu) \\
\mathcal{O}_{14} &= (d_\mu\mathcal{W}_\mu^+)(d_\nu\mathcal{W}_\nu^-)
\end{aligned}$$

    Corresponds to removing the operators L_{11}, L_{12}, L_{13} in usual basis
  - Using the EoM for the gauge fields one can remove:

$$\begin{aligned}
\mathcal{O}_{11} &= B_{\mu\nu}\mathcal{Z}_{\mu\nu} \\
\mathcal{O}_{12} &= \mathcal{Z}_{\mu\nu}\mathcal{Z}_{\mu\nu}
\end{aligned}$$

    Corresponds to removing the operators L_1, L_8 in usual basis
  - Similar procedure can be applied for source terms
- Only gauge-invariant source terms, no gauge-fixing in path integral $\rightarrow$ no gauge-artefacts can enter through EoM in our formalism

- The operators $\mathcal{O}_{15}, \ldots, \mathcal{O}_{18}, \mathcal{O}_3^s, \mathcal{O}_6^s, \mathcal{O}_7^s$ in $\mathcal{L}_4$ are proportional to terms in $\mathcal{L}_2$ $\rightarrow$ can remove these terms from basis by redefining parameters and LEC's in $\mathcal{L}_2$

- Summary
  The following parameters and LEC's will contribute to S-matrix elements (on-shell gauge bosons):
  - If we assume that $\bar{\rho} - 1 = \mathcal{O}(p^0)$:

    $\mathcal{L}_2: \; v, (\bar{\rho} - 1), g, g'$
    $\mathcal{L}_4: \; l_1, \ldots, l_8, l_{10}$ 9 low-energy constants

  - If we assume that $\bar{\rho} - 1 = \mathcal{O}(p^2)$:

    $\mathcal{L}_2: \; v, g, g'$
    $\mathcal{L}_4: \; l_1, \ldots, l_8, l_{10}, l_{16}$ 10 low-energy constants

- Note: Matter fields not included in analysis ! Results not directly comparable with literature, where L_{11}, L_{12}, L_{13} can be removed for massless external fermions:
  Appelquist, Wu (1993); Feruglio (1993); Dittmaier, Grosse-Knetter (1996)

5. Matching in the case of a heavy Higgs boson

- Example of a strongly interacting symmetry breaking sector
- Can evaluate LEC's in perturbation theory
  Longhitano (1981); Herrero, Morales (1994, 1995); Espriu, Matias (1995); Dittmaier, Grosse-Knetter (1996); Matias (1996)
- Problem: matching of gauge-dependent Green's functions of gauge bosons (off-shell) or integrating out in path integral $\rightarrow$ no guarantee that resulting $\mathcal{L}_{eff}$ and LEC's are gauge-independent
- We have redone calculation within our manifestly gauge-invariant functional approach
  Matching condition at low energies: $W_{eff} = W_{full}$
- Result up to one-loop in the Standard Model and up to $\mathcal{O}(p^4)$ in effective field theory:
  - Results for LEC's agree with literature, identical to ungauged case (where comparable)
  - Can be understood from counting: $g^2, g'^2 = \mathcal{O}(p^2)$ $\rightarrow$ gauge boson loops lead to corrections $\mathcal{O}(p^6)$ or to contributions proportional to $\mathcal{L}_2$ $\rightarrow$ can redefine parameters in $\mathcal{L}_2$

6. Conclusions

- Manifestly gauge-invariant approach to EW theory
  Nyffeler, Schenk, hep-ph/9812437

- Applied to EW chiral Lagrangian as the effective field theory description of a strongly interacting EW symmetry breaking sector

- Useful or even necessary to employ an effective field theory approach in such a case, in analogy to Chiral Perturbation Theory for QCD

- Presented well defined functional framework to study the following issues:

  – Number of independent parameters and low-energy constants in $\mathcal{L}_{eff}$: important to know for the phenomenological analysis $\leftrightarrow$ removing terms by using the equations of motion

  – Evaluating of low-energy constants of $\mathcal{L}_{eff}$ $\leftrightarrow$ matching of gauge-invariant Green's functions
    Example: SM with a heavy Higgs boson
    $\to$ obtain known results; main reason: momentum counting $g, g' = \mathcal{O}(p)$

- Outlook: Inclusion of fermions