Top Quark Production & Mass Definitions near Threshold

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(1) Renormalization group improvement; choice of scales

(2) · Top quark mass definitions; size of perturbative / long-distance effects

[Application at NNLO with A Signer and V A Smirnov.]

$$e^+e^- \longrightarrow \gamma^*, (Z^*) \longrightarrow t\bar{t} + X$$

$$\left[\hookrightarrow \quad b\bar{b}WW \right]$$

$$E \to E + i\Gamma_t$$

$$\sigma_{t\bar{t}} = \frac{4\pi \alpha_{em}^2}{3s} \cdot R$$

$$R = v \cdot \sum_k \left(\frac{\alpha_s}{v}\right)^k \cdot \left\{ 1 \text{ (LO)};\ \alpha_s, v \text{ (NLO)};\ \alpha_s^2, \alpha_s v, v^2 \text{ (NNLO)};\ \ldots \right\}$$

$$v = \left(\frac{\sqrt{s} - 2m_t}{m_t}\right)^{1/2}$$

[Hoang, Teubner; Melnikov, Yelkovsky; Yakovlev; MB, Signer, Smirnov]

There are potentially large logarithms:

$$\ln\left(\frac{m_t^2}{[m_t v]^2}\right) \approx 3.5 \qquad \ln\left(\frac{m_t^2}{[m_t v^2]^2}\right) \approx 9.5$$

[$m_t \sim 175$ GeV ; $m_t v \sim 30$ GeV ; $m_t v^2 \sim 1.5$ GeV]

$\Rightarrow$ sum also $(\alpha_s \ln v)^k$

$$R = v \cdot \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_\ell (\alpha_s \ln v)^\ell \cdot \left\{ 1 \text{ (LL)};\ \alpha_s, v \text{ (NLL)};\ \alpha_s^2, \alpha_s v, v^2 \text{ (NNLL)};\ \ldots \right\}$$

Only after summing logs can one determine the scale of α_s in various subprocesses consistently.

Summing logs requires the effective field theory approach.

Effective Theory Approach to $t\bar{t}$ Production

(a) hard quark and gluons: $k_0 \sim k_i \sim m_t$

(b) soft quark and gluons: $k_0 \sim k_i \sim m_t v$

(c) potential gluons: $k_0 \sim m_t v^2$, $k_i \sim m_t v$

(d) potential quarks: $k_0 \sim m_t v^2$, $k_i \sim m_t v$

(e) ultrasoft gluons: $k_0 \sim k_i \sim m_t v^2$

Integrate out (a), then (b+c):

$$\mathcal{L}_{\rm QCD}\left[Q(h,s,p),\ g(h,s,p,us)\right] \qquad \mu > m$$

$$\downarrow$$

$$\mathcal{L}_{\rm NRQCD}\left[Q(s,p),\ g(s,p,us)\right] \qquad mv < \mu < m$$

$$\downarrow$$

$$\mathcal{L}_{\rm PNRQCD}\left[Q(p),\ g(us)\right] \qquad \mu < mv$$

Resummation is automatic in PNRQCD perturbation theory.

The NRQCD Lagrangian and $\gamma^* t\bar{t}$ coupling at NNLL:

$$\begin{aligned}
\mathcal{L}_{\rm NRQCD} &= \psi^\dagger \left(iD^0 + \frac{\vec{D}^2}{2m} \right) \psi + \frac{1}{8m^3}\, \psi^\dagger \vec{D}^4 \psi \\
&- \frac{d_1\, g_s}{2m}\, \psi^\dagger \vec{\sigma} \cdot \vec{B} \psi + \frac{d_2\, g_s}{8m^2}\, \psi^\dagger \left(\vec{D} \cdot \vec{E} - \vec{E} \cdot \vec{D} \right) \psi \\
&+ \frac{d_3\, i g_s}{8m^2}\, \psi^\dagger \vec{\sigma} \cdot \left(\vec{D} \times \vec{E} - \vec{E} \times \vec{D} \right) \psi \\
&+ \sum_c \frac{d_{4c}\, g_s^2}{8m^2} \psi^\dagger \left[\kappa_c\right] \psi \sum_f \bar{q}_f \left[\kappa_c'\right] q_f \\
&+ \text{antiquark terms } \psi \to \chi \\
&+ \sum_c \frac{d_{5c}\, g_s^2}{8m^2} \psi^\dagger \left[\kappa_c\right] \psi\, \chi^\dagger \left[\kappa_c\right] \chi + \mathcal{L}_{\rm light}
\end{aligned}$$

$$\bar{Q}\gamma^i Q = c_1(\mu)\, \psi^\dagger \sigma^i \chi - \frac{c_2(\mu)}{6m^2}\, \psi^\dagger \sigma^i (i\vec{D})^2 \chi + \ldots$$

$$\begin{aligned}
c_1(m) &= 1 - \frac{8\alpha_s(m)}{3\pi} \\
&+ \frac{\alpha_s(m)^2}{(4\pi)^2} \left[-\frac{712}{27} - \frac{2044\pi^2}{81} - \frac{224\pi^2}{9} \ln 2 - \frac{2000\zeta(3)}{9} + \frac{176}{27}\, n_f \right] \\
&= 1 - 0.09 - 0.05 - \ldots \\
c_2(m) &= 1
\end{aligned}$$

All coefficients are defined in the $\overline{\rm MS}$ factorization scheme.

The PNRQCD Lagrangian:

$$\mathcal{L}_{\rm PNRQCD} = \psi^\dagger \left(i\partial^0 + \frac{\vec{\partial}^2}{2m} \right) \psi + \chi^\dagger \left(i\partial^0 - \frac{\vec{\partial}^2}{2m} \right) \chi$$
$$+ \int d^{d-1}r \left[\psi^\dagger \psi\right](r) \left(-\frac{C_F \alpha_s}{r} \right) \left[\chi^\dagger \chi\right](0)$$
$$+ \frac{1}{8m^3}\, \psi^\dagger \vec{\partial}^4 \psi - \frac{1}{8m^3}\, \chi^\dagger \vec{\partial}^4 \chi$$
$$+ \int d^{d-1}r \left[\psi^\dagger \psi\right](r)\, \delta V(r) \left[\chi^\dagger \chi\right](0)$$

with the $d = 4 - 2\epsilon$-dimensional heavy quark potential (running coupling terms omitted):

$$\delta\tilde{V}(\vec{p},\vec{q}) = -\frac{4\pi C_F \alpha_s}{\vec{q}^{\,2}} \Bigg[a_1 \frac{\alpha_s}{(4\pi)} + a_2 \frac{\alpha_s^2}{(4\pi)^2}$$
$$+ \frac{\pi \alpha_s |\vec{q}|}{4m} \left(\frac{\vec{q}^{\,2} e^{-\gamma_E}}{\mu^2} \right)^{-\epsilon} \frac{\Gamma(1/2-\epsilon)^2 \Gamma(1/2+\epsilon)}{\pi^{3/2}\Gamma(1-2\epsilon)}$$
$$\times \left(-\frac{C_F}{2}(1-2\epsilon) + C_A(1-\epsilon) \right)$$
$$+ \frac{\vec{p}^{\,2}}{m^2} + \frac{\vec{q}^{\,2}}{m^2} \left\{ d_1^2 \frac{d^2 - 7d + 10}{4(d-1)} - (1+d_2)\frac{1}{4} \right\} \Bigg]$$

PNRQCD describes $Q\bar{Q}$ pairs bound by the Coulomb interaction and interacting through (a) an instantaneous interaction (potential) and (b) ultrasoft gluons.

Power counting shows that (b) enters only at $\mathrm{N^3LO}$!

Leading logs:

There exist **no** leading logs

- $\psi^\dagger \sigma^i \chi$ has no 1-loop anomalous dimension
- $\psi^\dagger \left(iD^0 + \vec{D}^2/2m \right) \psi$ is not renormalized
- 1-loop correction to Coulomb potential (a_1) has no log

Next-to-leading logs:

Only from $c_1(\mu)\, \psi^\dagger \sigma^i \chi$; i.e. current renormalization

- Chromomagnetic, Darwin, .. interaction suppressed by v^2
- a_2 has no log ; non Coulomb potentials suppressed by $\alpha_s v$ and v^2

RGE: $$\mu \frac{d}{d\mu} c_1 = \left[0 \cdot \frac{\alpha_s}{\pi} + \gamma_1 \left(\frac{\alpha_s}{\pi} \right)^2 + \ldots \right] c_1$$

$\Rightarrow$ NLL
$$c_1(\mu) = 1 + \left[\left(c_1^{(1)} + S_1 \right) \frac{\alpha_s(m)}{\alpha_s(\mu)} - S_1 \right] \frac{\alpha_s(\mu)}{4\pi} + \ldots\ldots$$

($c_1^{(1)} \to -\frac{32}{3}$; $S_1 \to 560\pi^2/(27 b_0)$)

$$= 1 - \frac{8}{3} \frac{\alpha_s(m)}{\pi} + \left[-42.6 + 12.8 \ln \frac{m^2}{\mu^2} \right] \left(\frac{\alpha_s(m)}{\pi} \right)^2 + \ldots$$

scheme-dependent ($\overline{\text{MS}}$)

$\Downarrow$

absolute size of $(\alpha_s \log)^k$ more significant

near cancellation for top

The PNRQCD Lagrangian:

$$\mathcal{L}_{\rm PNRQCD} = \psi^\dagger \left(i\partial^0 + \frac{\vec{\partial}^2}{2m} \right) \psi + \chi^\dagger \left(i\partial^0 - \frac{\vec{\partial}^2}{2m} \right) \chi$$
$$+ \int d^{d-1}r \left[\psi^\dagger \psi\right](r) \left(-\frac{C_F \alpha_s}{r} \right) \left[\chi^\dagger \chi\right](0)$$
$$+ \frac{1}{8m^3}\, \psi^\dagger \vec{\partial}^4 \psi - \frac{1}{8m^3}\, \chi^\dagger \vec{\partial}^4 \chi$$
$$+ \int d^{d-1}r \left[\psi^\dagger \psi\right](r)\, \delta V(r) \left[\chi^\dagger \chi\right](0)$$

with the $d = 4 - 2\epsilon$-dimensional heavy quark potential (running coupling terms omitted):

$$\delta\tilde{V}(\vec{p}, \vec{q}) = -\frac{4\pi C_F \alpha_s}{\vec{q}^{\,2}} \left[a_1 \frac{\alpha_s}{(4\pi)} + a_2 \frac{\alpha_s^2}{(4\pi)^2} \right.$$
$$+ \frac{\pi \alpha_s |\vec{q}|}{4m} \left(\frac{\vec{q}^{\,2} e^{-\gamma_E}}{\mu^2} \right)^{-\epsilon} \frac{\Gamma(1/2-\epsilon)^2 \Gamma(1/2+\epsilon)}{\pi^{3/2}\Gamma(1-2\epsilon)}$$
$$\times \left(-\frac{C_F}{2}(1-2\epsilon) + C_A(1-\epsilon) \right)$$
$$\left. + \frac{\vec{p}^{\,2}}{m^2} + \frac{\vec{q}^{\,2}}{m^2} \left\{ d_1^2 \frac{d^2 - 7d + 10}{4(d-1)} - (1+d_2)\frac{1}{4} \right\} \right]$$

PNRQCD describes $Q\bar{Q}$ pairs bound by the Coulomb interaction and interacting through (a) an instantaneous interaction (potential) and (b) ultrasoft gluons.

Power counting shows that (b) enters only at N^3LO!

Consider $c_1(30\,\text{GeV})_{\text{NLL}}$, truncated at order α_s^k

k	in $\alpha_s(m)$	in $\alpha_s(\mu)$
0	1	1
1	0.91	0.88
2	0.96	1.01
3	0.97	0.97
4	0.98	0.98
5	0.98	0.98

(k = 1–2: fixed order NNLO; k = 3–5: NLL beyond NNLO)

⇒ NLL additional correction 4–6% $[c_1^2(\omega)]$ (depending on viewpoint); i.e. relatively small

Next-to-next-to-leading logs

Many sources: c_1, c_2, couplings in NRQCD Lagrangian, a_3, non-Coulomb potentials

$$d_1 = 1 + \frac{\alpha_s}{\pi}\left[-0.75\ln\frac{m^2}{\mu^2} + 2.2\right] + \dots$$

$$d_2 = 1 + \frac{\alpha_s}{\pi}\left[2.8\ln\frac{m^2}{\mu^2} + 1.5\right] + \dots$$

$$d_3 = 2d_1 - 1$$

[Manohar]

Here $\mu \sim mv^2$!

→ Order 1 modification of NNLO potentials possible

⇝ 10% effect ?

NNLL yet incomplete (work in progress). The following result is only NLL !

Top quark mass definitions

Claim: Sizeable shift is a mass renormalization artefact

Consider the (would-be) 1S toponium binding energy:

$$E(1S) \equiv M(1S) - 2m_{t,\text{pole}}$$

$$= -1.57\,\text{GeV}\left\{1 + [0.31 + 0.11] + [0.14 + 0.09 + 0.08] + \ldots\right\}$$

(annotations: 0.11: a_1; $0.31 + 0.11$ and 0.14: running coupling; 0.09: a_1, a_2; 0.08: non-Coulomb)

$$\Rightarrow \quad \delta E_{NNLO} \approx 500\,\text{MeV}\,!$$

Show:

(1) Poor convergence is related to large distances, i.e. contributions from $|\vec{q}| < m C_F \alpha_s$

(2) The long distance contributions can be absorbed into a mass redefinition. The new mass definition has better long-distance properties and smaller perturbative corrections.

$$V_{static}(r) = -\frac{C_F \alpha_s}{r}\left[1 + \mathcal{O}(\alpha_s) + \Lambda_{QCD}\, r + \ldots\right]$$

- Linear power correction from $\int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}}\, \tilde{V}(\vec{q})$ at $\vec{q}\cdot\vec{r} \ll 1$, not from $\tilde{V}(\vec{q})$
- Large power correction $\Rightarrow$ large perturbative coefficient from loop momentum $\Lambda_{QCD} \ll \vec{q} \ll 1/r$ [renormalon argument]

Define a subtraction

$$\delta m(\mu_f) = -\frac{1}{2}\int_{|\vec{q}|<\mu_f} \frac{d^3\vec{q}}{(2\pi)^3}\,\tilde{V}(\vec{q}) \qquad \Lambda_{QCD} \ll \mu_f < mv$$

$$V^{sub}(r) = V(r) + 2\delta m(\mu_f)$$

$$m_{PS}(\mu_f) = m_{pole} - \delta m(\mu_f)$$ "potential-subtracted (PS) mass"

then

$$-\frac{\nabla^2}{m_{pole}} - V(r) - (\sqrt{s} - 2m_{pole}) = -\frac{\nabla^2}{m_{pole}} - V^{sub}_{(r,\mu_f)} - (\sqrt{s} - 2m_{PS}(\mu_f))$$

large perturbative corrections removed

Check this by computing 1S energy shift due to

$$\delta\tilde{V}_k(\vec{q}) = -\frac{4\pi C_F \alpha_s}{\vec{q}^2}\left(-\frac{b_0\alpha_s}{4\pi}\ln\frac{\vec{q}^2}{\mu^2}\right)^k$$

$$\delta E_k = \int \frac{d^3\vec{q}}{(2\pi)^3}\frac{d^3\vec{p}}{(2\pi)^3}\,\psi^*_{1S}(\vec{p}+\tfrac{\vec{q}}{2})\,\delta\tilde{V}_k(\vec{q})\,\psi_{1S}(\vec{p}-\tfrac{\vec{q}}{2}) = \int\frac{d^3\vec{q}}{(2\pi)^3}\frac{1}{\left(1+\frac{\vec{q}^2}{(mC_F\alpha_s)^2}\right)^2}\,\delta\tilde{V}_k(\vec{q})$$

$$2\delta m(\mu_f) = \int\frac{d^3\vec{q}}{(2\pi)^3}\,(-1)\,\delta\tilde{V}_k(\vec{q})$$

$$M_{1S} = 2m_{pole} + E_0 + \delta E_k = 2m_{PS}(\mu_f) + E_0'(\mu_f) + (\delta E_k(\mu_f) + 2\delta m(\mu_f))$$

k	1	2	3	4	5	6	
δE/MeV	−488	−214	−109	−78	−67	−70	...
$(\delta E + 2\delta m)$/MeV	+97	+0.8	+3.5	+0.1	+0.3	0	

$\mu = 30\,\text{GeV}$, $\mu_f = 20\,\text{GeV}$

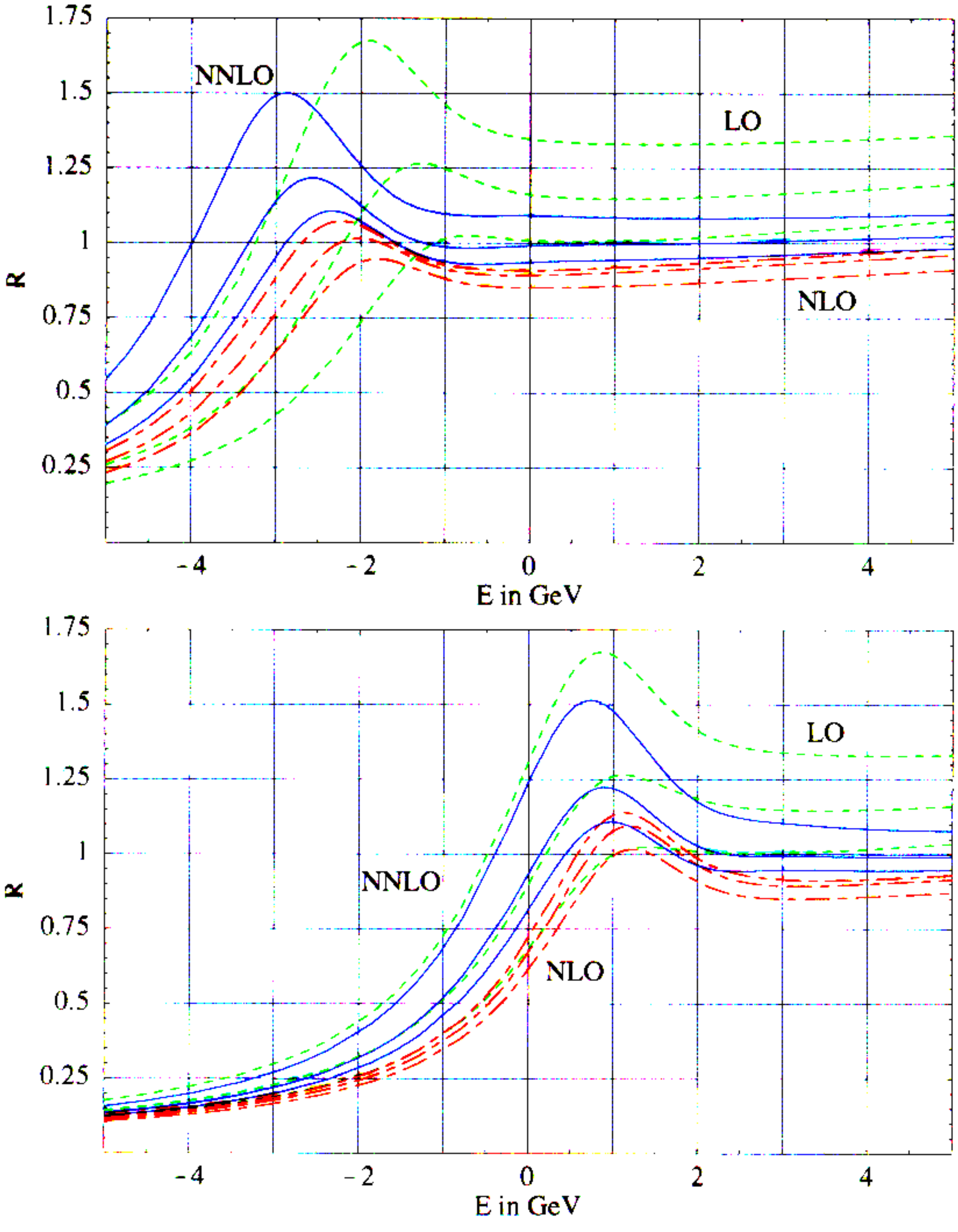
⇒ cancellation of large energy shifts

Crucial: We have to show that m_{PS} does not re-introduce the large corrections when it is used in another observable

The NNLO/NLL cross section in the pole and potential subtraction (PS mass) scheme.

Upper panel: Pole scheme.

Lower panel: PS scheme.



Parameters: $m_t = 175\,\mathrm{GeV}$, $\Gamma_t = 1.40\,\mathrm{GeV}$, $\alpha_s(M_Z) = 0.118$, scale of α_s equal to $\{1/2, 1, 2\} \cdot 30\,\mathrm{GeV}$

The pole mass has itself large perturbative corrections when it is related to $m_{\overline{MS}}$ (or another IR-insensitive definition) [MB, Braun, Bigi et al.]

$$m_{pole} - m_{\overline{MS}} \propto \alpha_s \int d^4k \, \frac{m}{k^2\left(k^2+2p\cdot k+(p^2-m^2)\right)}$$

never zero for top ($\langle p^2-m^2\rangle \sim m\Gamma_t$) but assumed zero in def of m_{pole}!

$$\longrightarrow m\left[\mathcal{O}(\alpha_s)+\dots+\frac{\Lambda_{QCD}}{m}\right]$$

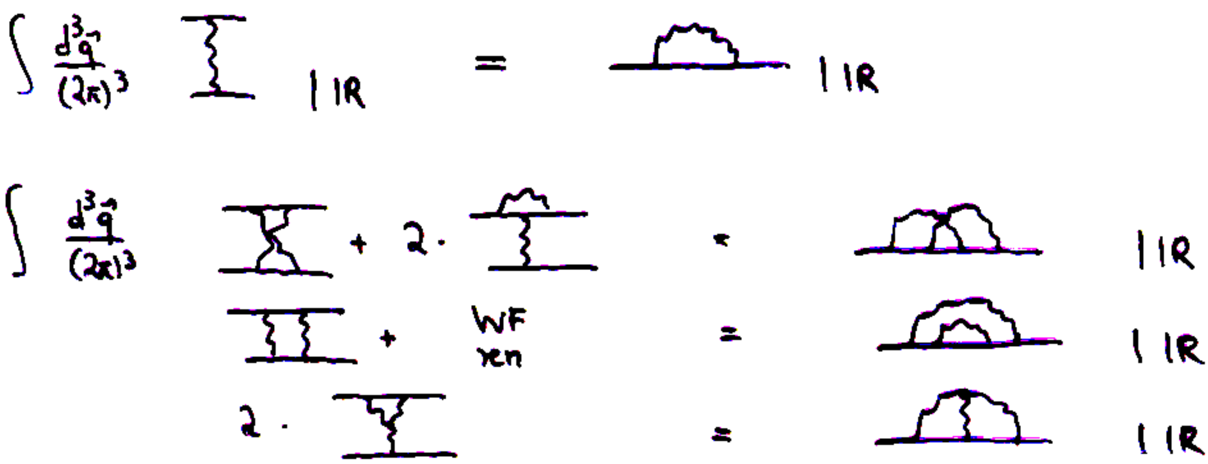
$$m_{PS} - m_{\overline{MS}} = \left[m_{pole}-m_{\overline{MS}}\right] - \delta m(\mu_f)$$

$\frac{\Lambda_{QCD}}{m}$ cancels

$$\longrightarrow m\left[\mathcal{O}(\alpha_s)+\dots+\frac{\Lambda_{QCD}^2}{m^2}+\dots\right]$$

[MB; Hoang et al.]

Sketch:



3 loops: ??

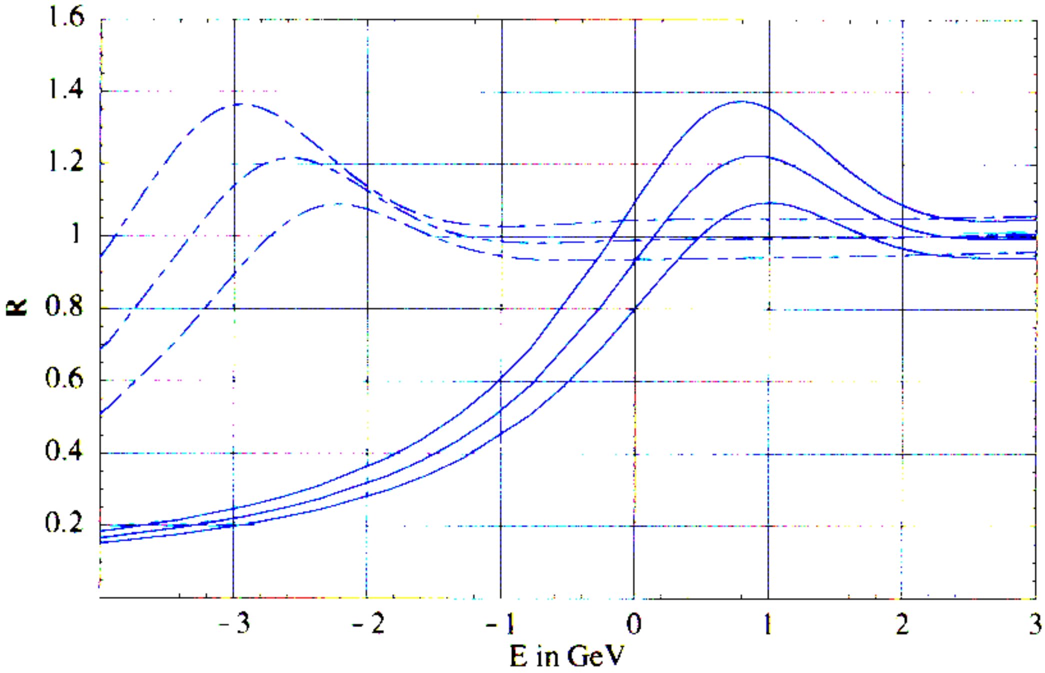
In numbers:

$$m_{t,pole} = [165.0 + 7.6 + 1.6 + 0.6\ (\text{est})]\ \text{GeV}$$

$$m_{t,PS(20\,\text{GeV})} = [165.0 + 6.7 + 1.2 + 0.3\ (\text{est.})]\ \text{GeV}$$

$[\, m_{t,\overline{MS}}(m_t) = 165\ \text{GeV} \,]$

$m_t - \alpha_s$ correlation (NNLO):



Solid: PS scheme.

Dashed: pole scheme.

Parameters as above.