New Results on Top-Antitop Production close to Threshold

André H. Hoang
CERN

ECFA, Oxford 20.03.99

based on AHH., Thomas Teubner hep-ph/9904.

$\Rightarrow$ tremendous progress in the theoretical description of $t\bar{t}$ production close to threshold within the last 1½ years

$\downarrow$

NNLO QCD relativistic corrections understood

$\swarrow$ $\sigma^{tot}(e^+e^- \xrightarrow{\gamma} t\bar{t})$ $\qquad\qquad$ $\searrow$ EFT

AHH, Teubner $\rightarrow$ [Orsay, Lund]
Melnikov, Yelkovski; Yakovlev
Beneke, Signer, Smirnov (NLL)

Lepage, Caswell, et.al, NRQCD
Pineda, Soto, PNRQCD

Tools: threshold expansion (Beneke, Smirnov)
direct matching (AHH)

Misc. results: axial ($\gamma\gamma$) (Penin, Pivovarov)
axial (Kühn, Teubner)

- $\rightarrow$ effective field theoretical tools proven to be most economic framework; conceptually solid tool box
- $\rightarrow$ a number of new problems have been revealed:
  mass, normalization of σ_{tot}
  $\hookrightarrow$ need to be addressed, solved
- $\rightarrow$ observables beyond σ_{tot} need to be tackled, $\frac{d\sigma}{dx}$
- $\rightarrow$ conceptually deeper understanding is required
  - EW effects
  - gauge invariance and off-shellness
  - interconnection
- $\rightarrow$ N^3LO: How large are effects?

Content of this talk

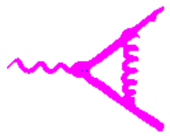
- Why to use an effective theory for $t\bar{t}$ production close to threshold.
- Effective field theory $\longrightarrow$ NRQCD
- Our method $\rightarrow$ first results in the pole mass scheme (for the last time ...)
- Top mass definition / determination
- Normalization of the total cross section
- Conclusion

... lot's of plots ...

Why are $\mathcal{O}(v^2)$, NNLO corrections to σ_{tot}^{thresh} non trivial ?

- Conventional pert. theory in α_s fails

$V(r) \sim -\frac{C_F \alpha_s}{r}$, $v \ll 1$

 $\sim \frac{\alpha_s}{v} \sim \mathcal{O}(1) \sim$ Born ← IR problem

→ resummation of $(\alpha_s/v)^n$ terms to all orders in α_s required already in lowest order approximation [non relativ. limit]

→ simultaneous expansion in α_s and v , $\frac{\alpha_s}{v} \sim 1$

→ Schrödinger theory

- Schrödinger theory fails → $\left(-\frac{\vec\nabla^2}{m} - V(\vec r) - E_n\right)\psi_n(\vec r) = 0$

$R = \frac{\sigma_{t\bar t}}{\sigma_{pt}} \sim \mathrm{Im} \sum_n \frac{|\psi_n(0)|^2}{E_n - E - i\Gamma}$ ⟹ How to get beyond LO? How to calculate relativ. corr. ?

Darwin term: $H_D = \frac{C_F \alpha_s \pi}{m^2} \delta^{(3)}(\vec r)$

→ $\delta E_n = \langle n | H_D | n \rangle = \frac{C_F \alpha_s \pi}{m^2} |\psi_n(0)|^2$

→ $\delta |n\rangle = \sum_{\ell \neq n} \frac{|\ell\rangle\langle \ell|}{E_n - E_\ell} H_D |n\rangle$

$\delta\psi_n(0) = \left[\lim_{E\to E_n} \left(G^c(0,0) - \frac{|\psi_n(0)|^2}{E_n - E}\right)\right] \frac{C_F \alpha_s \pi}{m^2} \psi_n(0)$

↑ UV divergent

$G^c(0,r) \xrightarrow{r\to 0} \sim \frac{1}{r}, \ln r$ ← UV problem

$G^{free}(0,r) = \frac{m}{4\pi} \frac{1}{r} e^{i\sqrt{mE}\, r}$

⟹ Bethe-Salpeter: fund. equations of Green fct's in full QCD → "work it out yourself !"

⟹ NRQCD: Lagrangian formulation, renormalization; systematics by formalized tool box; power-counting manifest → "share work with the formalism!"

Systematics → EFT

$$\text{QCD}_{+\,\text{ew}} \qquad \text{3 scales:} \quad \underset{\text{"hard"}}{M}\,, \quad \underset{\text{"soft"}}{M\langle v\rangle}\,, \quad \underset{\text{"ultrasoft"}}{M\langle v\rangle^2}$$

↓ integrate out hard gluons + quarks

NRQCD (Caswell, Lepage)

$$\mathcal{L}_{\text{NRQCD}} = \psi^+ \left[i D^0 + \frac{\vec{D}^2}{2M} + \frac{\vec{D}^4}{8M^3} + \ldots \right] \psi$$

$$+ \psi^+ \left[c_F \frac{g}{2M} \vec{\sigma}\vec{B} + c_D \frac{g}{8M^2} (\vec{D}\vec{E} - \vec{E}\vec{D}) + c_S \frac{g}{8M^2} i\vec{\sigma}(\vec{D}\times\vec{E} - \vec{E}\times\vec{D}) \right] \psi$$

$$+ \chi^+\chi + \text{higher dim.} + \text{light} \qquad + \text{operators with complex coeff. (ew. effects)}$$

↓ integrate out soft gluons

PNRQCD (Pineda, Soto)

Labelle
Grinstein, Rothstein
Manohar, Luke, Savage
Grießhammer
Beneke, Smirnov

$$\mathcal{L}_{\text{PNRQCD}} = \tilde{\mathcal{L}}_{\text{NRQCD}}$$

$$+ \int d^3\vec{r}\, [\psi^+ T^A \psi](\vec{r}) \left[-\frac{\alpha_s}{r}\left[1 + \frac{\alpha_s}{\pi}\ldots\right] + V_{3F}(\vec{r}) - \frac{C_A C_F \alpha_s^2}{2M r^2} \right] [\chi^+ T^A \chi](0)$$

$$+ \text{light}$$

"Multipole expansion" of $Q\bar{Q}G$ vertices in $\tilde{\mathcal{L}}_{\text{NRQCD}}$

$M\langle v\rangle^2 > \Lambda_{\text{QCD}}$ ⇒ retardation effects are NNNLO

→ $Q\bar{Q}$ at NNLO described by Schrödinger theory with inst. potentials

$$R_{t\bar t}^{thr} = \frac{\sigma(e^+e^- \to t\bar t^{\,\gamma})^{thr}}{\sigma_{pt}}$$

NRQCD

$$= \frac{\pi Q_t^2}{m_t^2}\, C_1\, \mathrm{Im}\left[\langle 0|(\psi^\dagger \vec\sigma \chi)(\chi^\dagger \vec\sigma \psi)|0\rangle\right]$$

$$+ \frac{\pi Q_t^2}{m_t^4}\, C_2\, \mathrm{Im}\left[\langle 0|(\psi^\dagger \vec\sigma \chi)(\chi^\dagger \vec\sigma (-\tfrac{i}{2}\overleftrightarrow{D})^2 \psi) + \text{h.c.}\, |0\rangle\right] + \ldots$$

NRQCD matching

$\mathrm{Im}\, G(0,0)^{reg}$

<u>NNLO Schrödinger equation</u> (neglect subtleties with ew. corr.)

$$\left(-\frac{\vec\nabla^2}{m_t} - \frac{\vec\nabla^4}{4m_t^3} + V(\vec r) - (\sqrt{s} - 2m_t) - i\Gamma_t\right) G(\vec r, \vec r\,') = \delta^{(3)}(\vec r - \vec r\,')$$

"NNLO" means in σ^{tot}: $\quad v \sum\limits_{n=0}^{\infty} \left(\frac{\alpha_s}{v}\right)^n \times \left[\, 1 \;;\; \alpha_s, v \;;\; \alpha_s^2, \alpha_s v, v^2 \,\right]$

LO $\qquad$ NLO $\qquad$ NNLO

$$V = V_{\text{Coulomb}} + V_{\text{Breit-Fermi}} + V_{\text{non-abelian}}$$

Fischler, Billoire, Schröder, Peter $\qquad$ "QED" $\qquad$ from soft gluons + quarks

<u>Our Method</u>

→ Solve the Schrödinger eq. in momentum space on a momentum grid

→ Regularization scheme: cut-off $\Lambda \sim O(m_t)$

α_s defined in $\overline{MS}$

→ allows for <u>exact</u> solution of the Schrödinger eq.

- gauge invariance
- power counting breaking effects
- routing dependence

⟶ cured by matching up to terms of order α_s^3

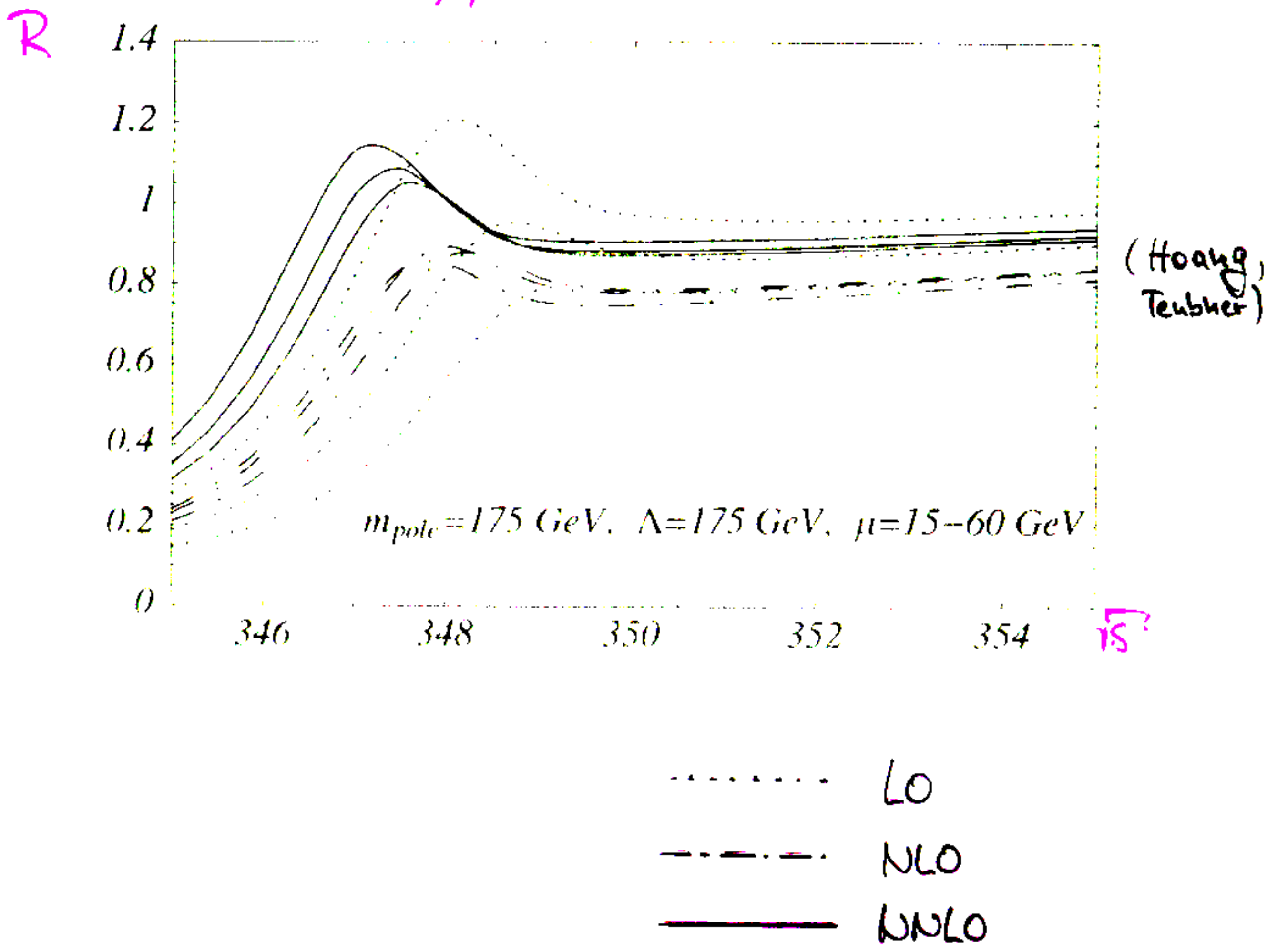
First Results in the

Pole Mass Scheme

for the last time ....

Total Cross Section

$$R = \frac{\sigma(e^+e^- \xrightarrow{\gamma} t\bar{t} \rightarrow W^+W^- b\bar{b})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



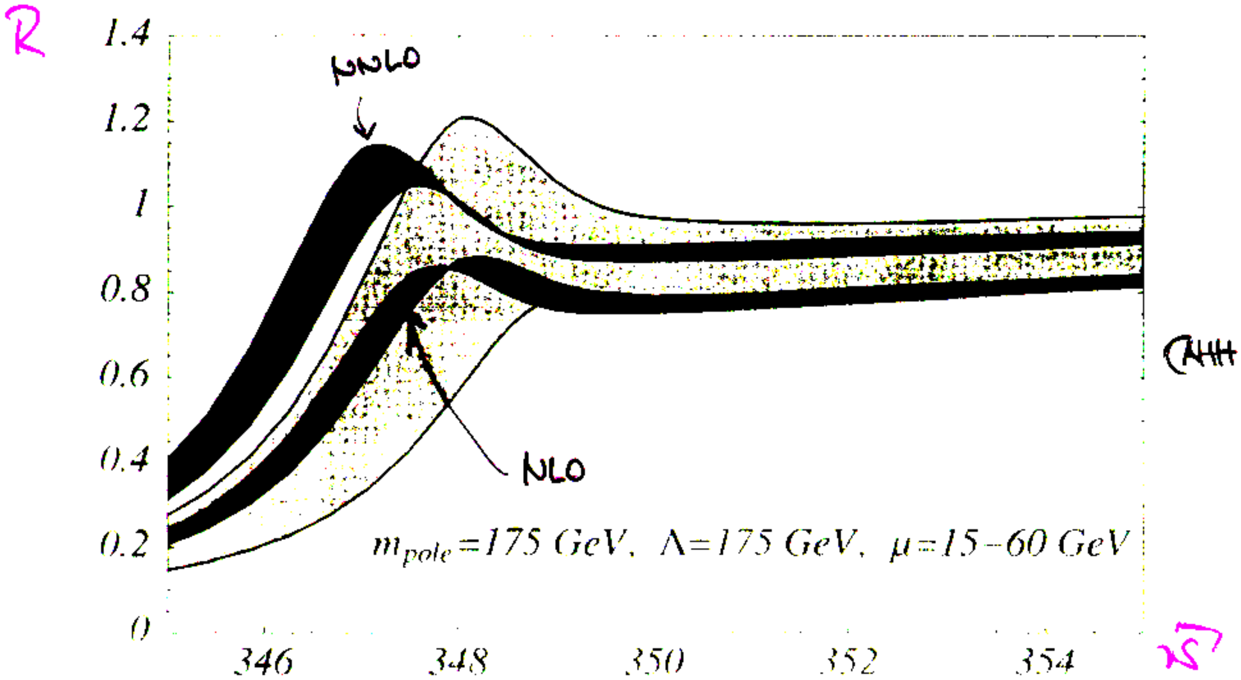
→ renormalization scale dependence ←

15 GeV < μ < 60 GeV

We observe: • no reduction from NLO to NNLO

• instability of peak position

• NNLO corrections large (peak + normalization) same picture ...



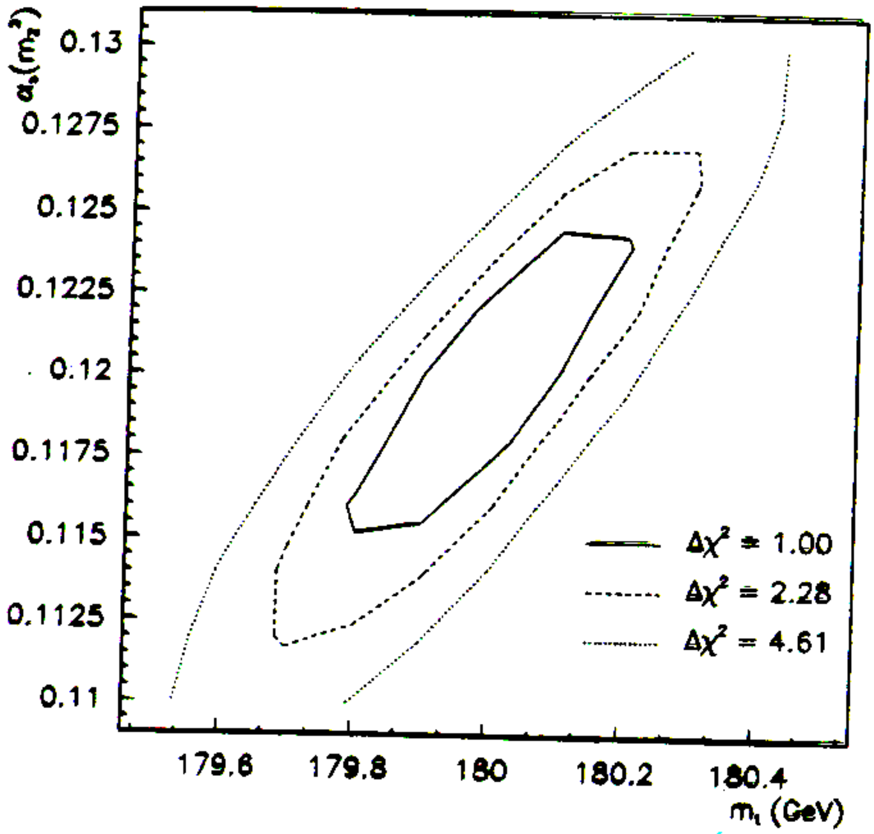
(AHH, Teubner)

light grey	LO
grey	NLO
dark	NNLO

→ strong dependence of peak position

$M_{peak} \sim 2m_{pole} - \frac{m_{pole}(C\alpha_s)^2}{4} + \dots$

leads to sizeable correlation $m_{pole} \leftrightarrow \alpha_s$

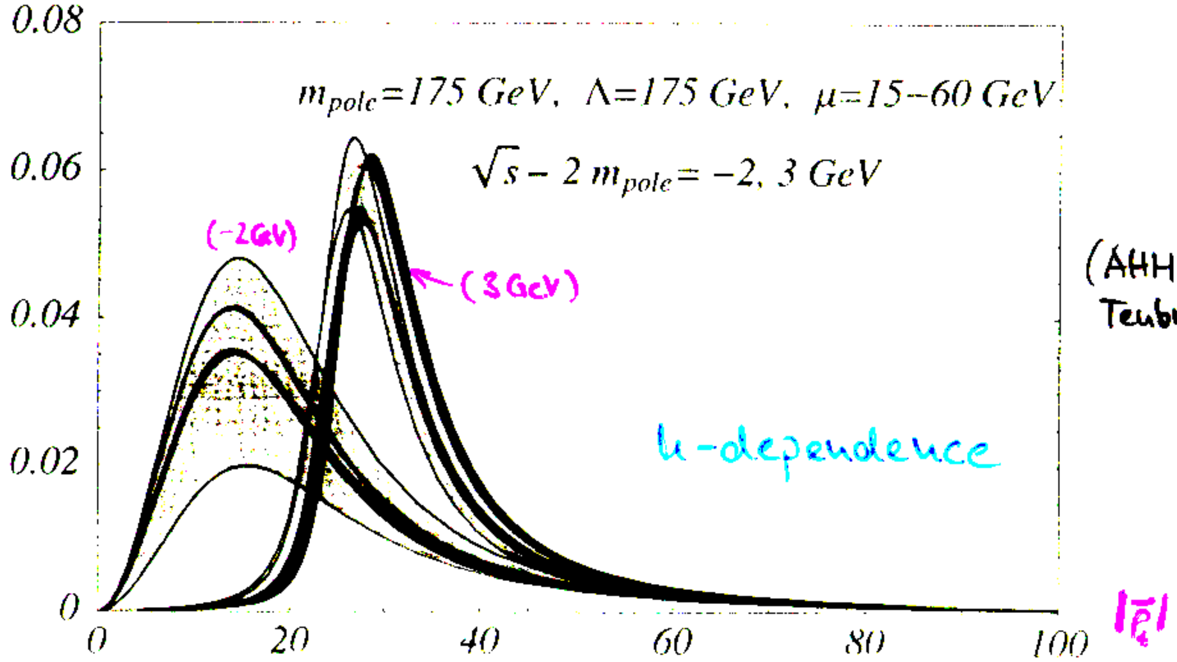


Results of the χ^2 fit using the total cross section and the momentum distribution.

Comas, Miguel, Martinez, Orteu

The 3-Momentum Distribution (pole mass scheme)

$$\frac{d\sigma_\gamma}{d|\vec{p}_t|} \sim |G(E,p)|^2 \, \Gamma_t \qquad \sigma_\gamma = \sigma(e^+e^- \to t\bar{t})$$



(AHH Teubner)

light grey	LO
grey	NLO
dark	NNLO

→ interconnection + axial-vector interference + axial-axial ← contributions lead to additional corrections

we observe:
- NNLO corrections large
- instability of peak position

Top Quark Mass Determination [from $\sigma_{\rm total}$!]

→ let's simplify the situation: neglect beamstrahlung
  ⇝ top mass determination depends on peak position in $\sigma_{\rm tot}$

→ we have seen problems in the pole scheme:

- large corrections to peak position
- large correlation: $m^{\rm pole} \longleftrightarrow \alpha_s$

[Beneke et al. Figi et al.]

⇝ Does this all come frome the $O(\Lambda_{QCD})$ ambiguity in the pole mass?

Origin of large corrections

$$M_{\substack{\rm peak\\ \rm pk}} = 2m_t^{\rm pole} - \delta M_{\rm pk}^{\rm LO} - \delta M_{\rm pk}^{\rm NLO} - \delta M_{\rm pk}^{\rm NNLO,\,large\,\beta_0} - \delta M_{\rm pk}^{\rm NNLO,\,rest}$$

	$\delta M_{\rm pk}^{\rm LO}$	$\delta M_{\rm pk}^{\rm NLO}$	$\delta M_{\rm pk}^{\rm NNLO,\beta_0}$	$\delta M_{\rm pk}^{\rm NNLO,rest}$
[GeV]	1.26	0.75	0.31	0.27

$\mu = 30$ GeV
$\alpha_s(M_Z) = 0.118$
$m_{\rm pole} = 175$ GeV

most sensitive to the IR

→ large corrections come from dominant IR sensitive terms
⊕ subleading IR, relativistic corrections

⇝ corrections only partly because of IR sensitivity in pole mass

Origin of large correlations

$\mu = 30$ GeV, $m_{\rm pole} = 175$ GeV

$\alpha_s(M_Z)$	$\delta M_{\rm pk}^{\rm LO}$	$\delta M_{\rm pk}^{\rm NLO}$	$\delta M_{\rm pk}^{\rm NNLO}$
0.114	1.07	0.71	0.52
0.118	1.26	0.75	0.58
0.122	1.44	0.80	0.64

[GeV]

⇝ correlation nothing to do with IR sensitivity of pole mass

Correlation dominated by LO / nonrelativistic limit

$\Longrightarrow$ Requirements to a proper "threshold mass"

(A) short-distance mass $\rightarrow$ mass definition without $O(\Lambda_{QCD})$ ambiguity

(B) stability of peak position $\rightarrow$ small corrections
small correlations to α_s, μ, Γ

$\Rightarrow$ optimal choice : M_{1S} - $\frac{1}{2}$ fictitious $1\,{}^3S_1$ toponium mass

$$\boxed{M_{1S} \equiv \frac{1}{2}\langle \psi_{1S} | H | \psi_{1S} \rangle}$$

(A) Mass M_{1S} is a short-distance mass.

[AHH, Smith, Stelzer, Willenbrock
Beneke
AHH, Ligeti, Manohar]

$$M_{1S}^{\text{NNLO}} = \frac{1}{2}\int \frac{d^3\vec p}{(2\pi)^3}\frac{d^3\vec q}{(2\pi)^3}\,\tilde\varphi_{1S}(\vec p)\,H(\vec p,\vec q)\,\tilde\varphi_{1S}(\vec q)$$

$$\overset{\text{IR}}{\sim}\ m_{\text{Pole}} + \frac{1}{2}\int \frac{d^3\vec p}{(2\pi)^3}\frac{d^3\vec q}{(2\pi)^3}\,|\tilde\varphi_{1S}(\vec p)|^2\,V_c(\vec p-\vec q)$$

$$= m_{\text{Pole}} + \frac{1}{\pi^2}\int_0^{\mu_f} dq\, V_c(q) \qquad \text{q.e.d.}$$

$|\vec p - \vec q| < \mu_f \ll m_t \alpha_s$

(B) stability achieved by construction.

$M_{\text{peak}} - 2M_{1S} \sim \Gamma_t$ $\leftarrow$ weakly dependent on α_s, μ

$\rightarrow$ M_{1S} very close to a physical observable

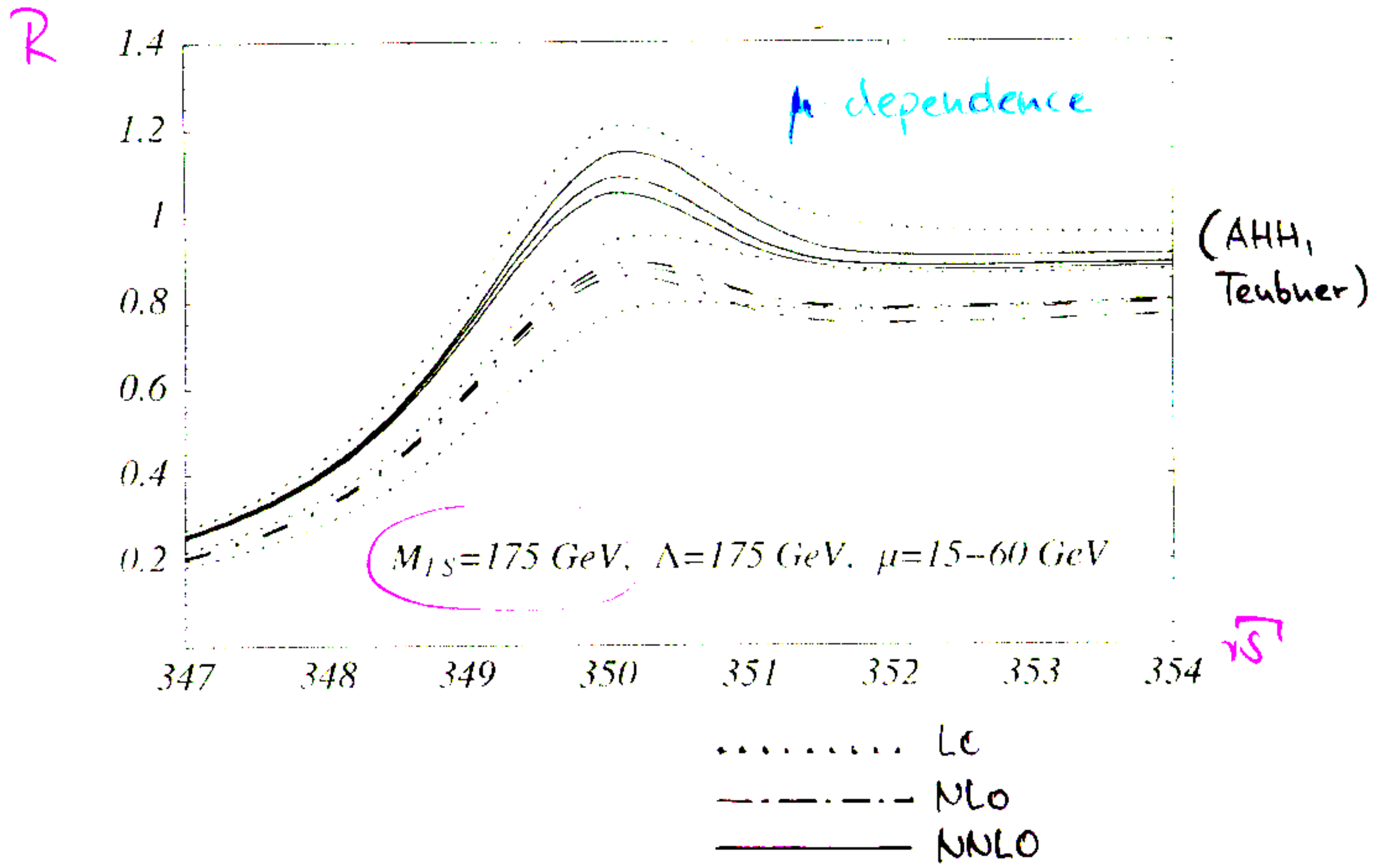
$\rightarrow$ order independent (almost)

$\rightarrow$ almost correlation-free $\Longrightarrow$ small uncertainties in mass extraction

$\delta M_{1S}^{\text{theoretical}} \lesssim 100$ MeV

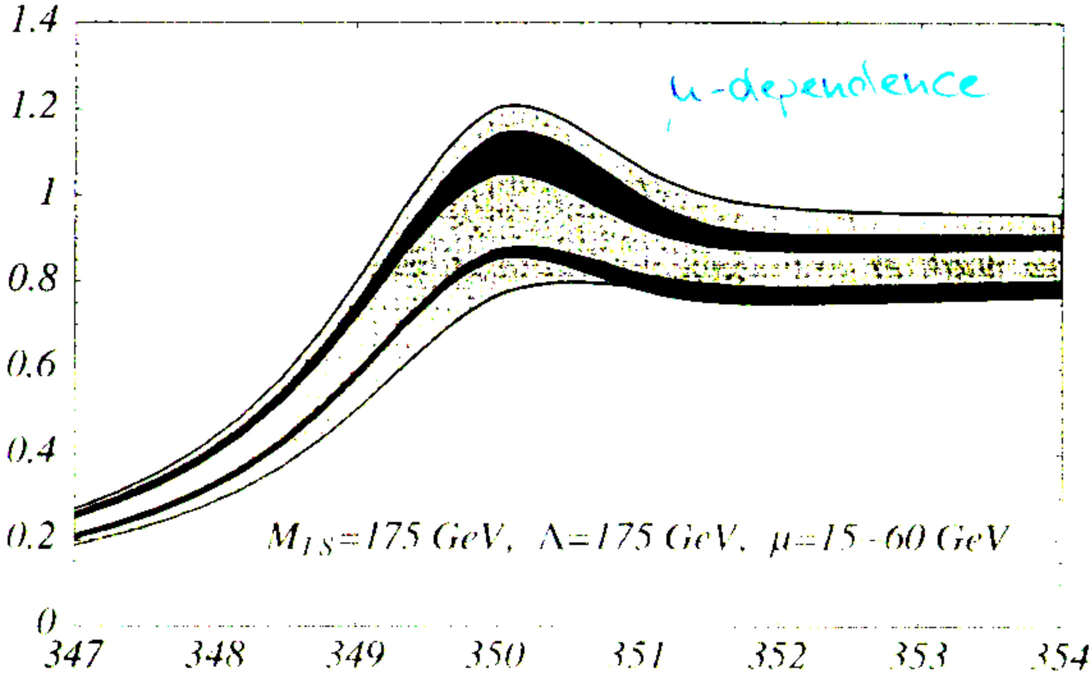
$\rightarrow$ Fig M_{1S}, α_s/μ
Fig M_{1S} correlation
Fig M_{1S} distribution

Total Cross Section in the 1S Scheme



same picture ... σ_{tot} in the 1S scheme

R



$\sqrt{s}$

(AHH, Teubner)

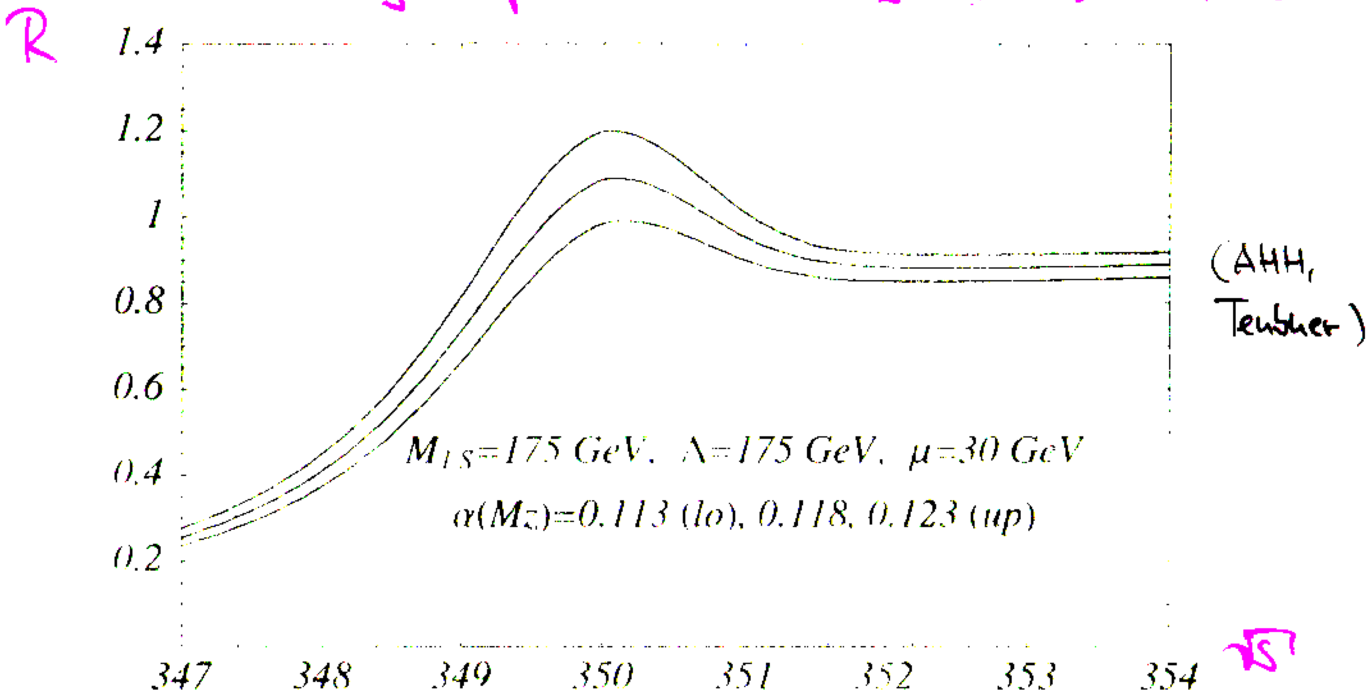
light grey	LO
grey	NLO
dark	NNLO

We observe:

- peak position stable (with respect to μ)
- corrections to the normalization still large

1S - scheme

α_s -dependence $\alpha_s(M_Z) = 0.118 \pm 0.005$

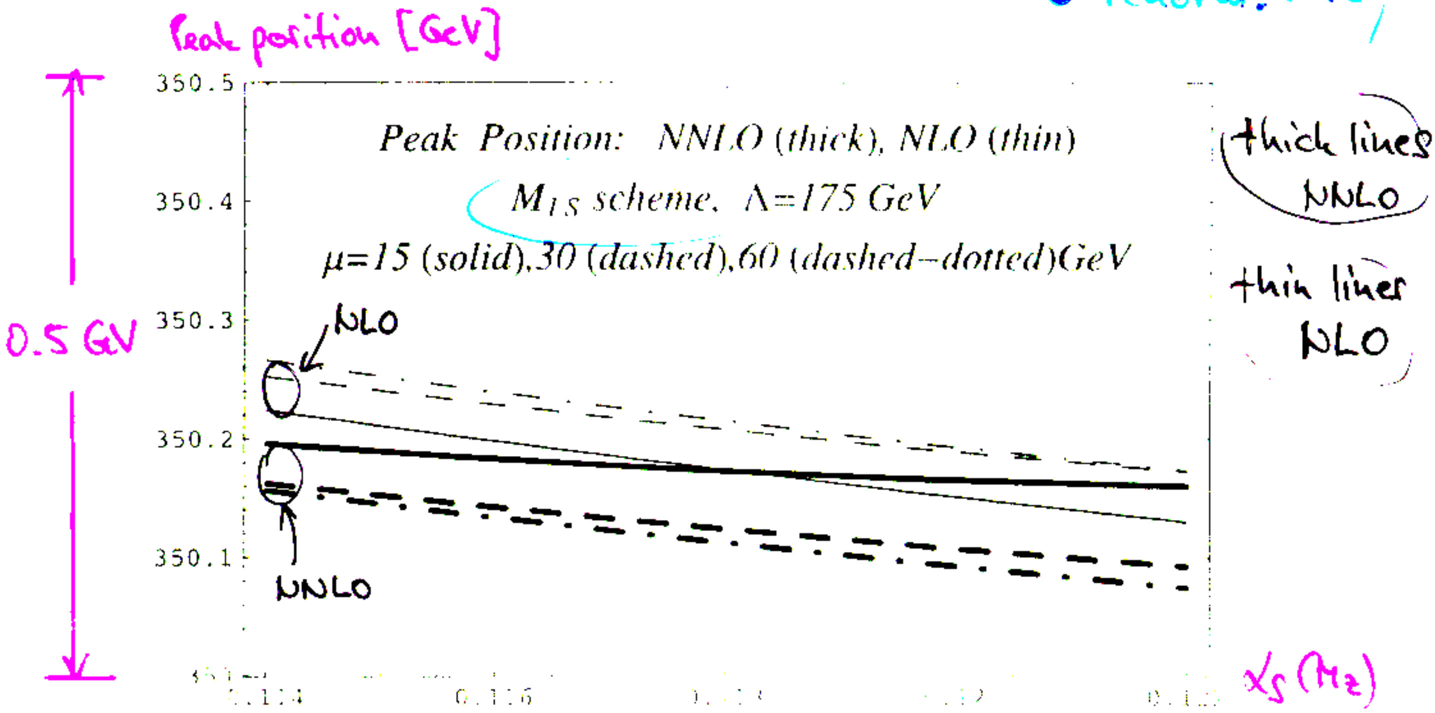


We observe:

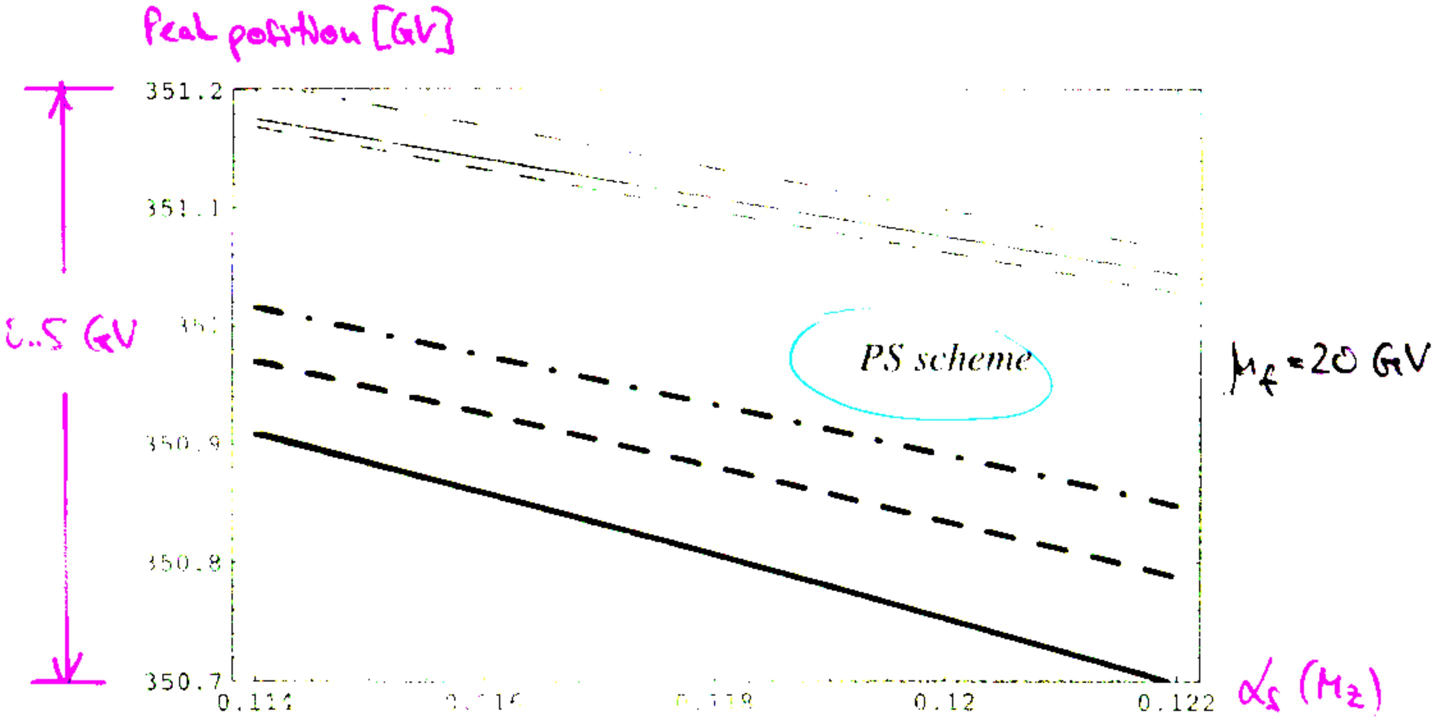
- peak position stable (with respect to α_s)

  → we can expect that the correlation mass ⟺ α_s which was visible in the pole scheme is reduced dramatically

Dependence of the peak position on
- order
- $\alpha_s(M_Z)$
- renorm. scale μ



$\rightsquigarrow$ $\delta M_{1S}^{\text{theoretical}} \lesssim 100$ MeV [$\delta M_{1S} \sim \frac{1}{2}\,\delta M_{peak}$]



$\rightsquigarrow$ $\delta M_{PS}(\mu_f = 20\,\text{GeV}) \approx 200$ MeV

$\Longrightarrow$ theoretical uncertainties larger in the PS scheme than in the 1S scheme

Where are the problems we had in the pole scheme gone?

→ Do we have to pay a price later?? → Answer: NO!

- potentially large IR sensitive corrections are cancelled because they are universal
  ($\leftrightarrow$ σ_{tot} is theoretically "better" defined than the pole mass)
- correlation-free parameters lead to reduced uncertainties [→ M_{1S} is "almost" a physical observable]

What can one do with M_{1S}?

- use it directly as a mass definition
- relate it to the $\overline{MS}$ mass

example: $M_{1S} = 175 \pm 0.15\,\text{GeV}$

$\alpha_s(M_Z) = 0.118 \pm x \cdot 0.001$

$$\Rightarrow \overline{m}_t(\overline{m}_t) = [175 - 7.58 - 0.96 - \underset{\uparrow}{0.23} \pm \underbrace{0.15}_{\delta M_{1S}} \pm \underbrace{x\, 0.07}_{\delta\alpha_s(M_Z)}]\ \text{GeV}$$

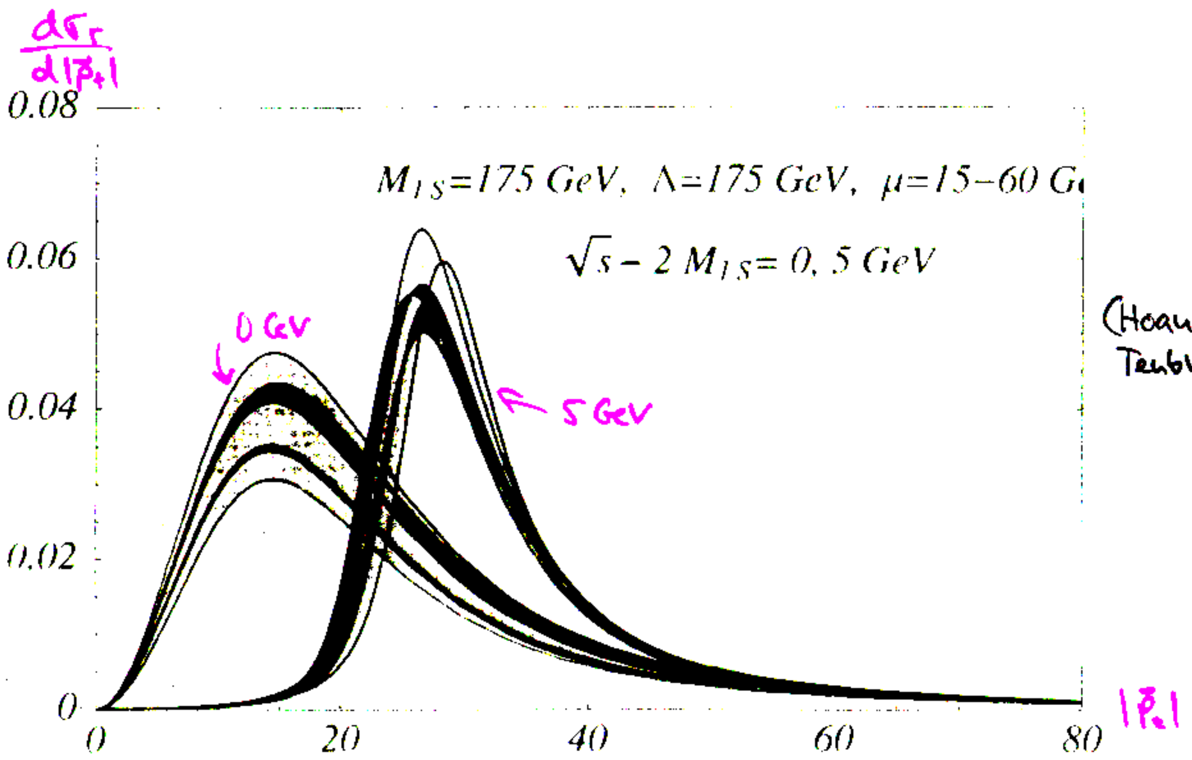
$O(\alpha_s^3)$ large β_0 for $(\overline{m}(\overline{m}) - m_{pole})$

→ 3 loop $(\overline{m}(\overline{m}) - m_{pole})$: $\delta\overline{m}(\overline{m}) \approx \delta M_{1S}$ $(x=1)$

2 loop $(\overline{m}(\overline{m}) - m_{pole})$: $\delta\overline{m}(\overline{m}) \approx 300\,\text{MeV}$ $(x=1)$

without $\frac{d\sigma}{d|\vec{p}_t|}$!

Momentum Distribution in the 1S scheme



(Hoang, Teubner)

light grey LO
grey NLO
dark NNLO

We observe: • stability of peak position not improved in 1S scheme

→ Why?

Answer: peak in σ_{tot} is not related to peak in $\frac{d\sigma}{d|\vec{p}_t|}$

$E_{c.m.} = \sqrt{s}$ $\qquad |\vec{p}_t|$

related to color reconnection problem

→ instability of $\frac{d\sigma}{d|\vec{p}|}$ cannot be eliminated by mass redefinition because $|\vec{p}_{top}|$ is ambiguous itself (top is a color triplet)

→ it works for σ_{tot} because the c.m. energy is well defined.

The normalization of the cross section

→ normalization not affected by choice of mass ⟶ shift in energy

→ $\mathcal{O}(20\%)$ NNLO corrections ⟷ $\mathcal{O}(v^2, \alpha_s^2)$?

⤳ NO CONVERGENCE OR ACCIDENTAL?

Origin of large corrections

$$\sigma_{tot} \sim \mathrm{Im} \sum_n \frac{|\psi_n(0)|^2}{E_n - E - i\Gamma}$$

Due to sensitivity to the IR ?

$$\delta \psi_n(0) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{d^3\vec{q}}{(2\pi)^3} \sum_{\ell \neq n} \frac{\psi_\ell(0)\, \psi_\ell^*(\vec{p})}{E_\ell - E_n}\, \delta H(\vec{p},\vec{q})\, \psi_n(\vec{q}) \qquad |\vec{p}-\vec{q}| < \mu_f \ll m\alpha_s^2$$

$\underset{\sim}{\mathrm{IR}}$ 0

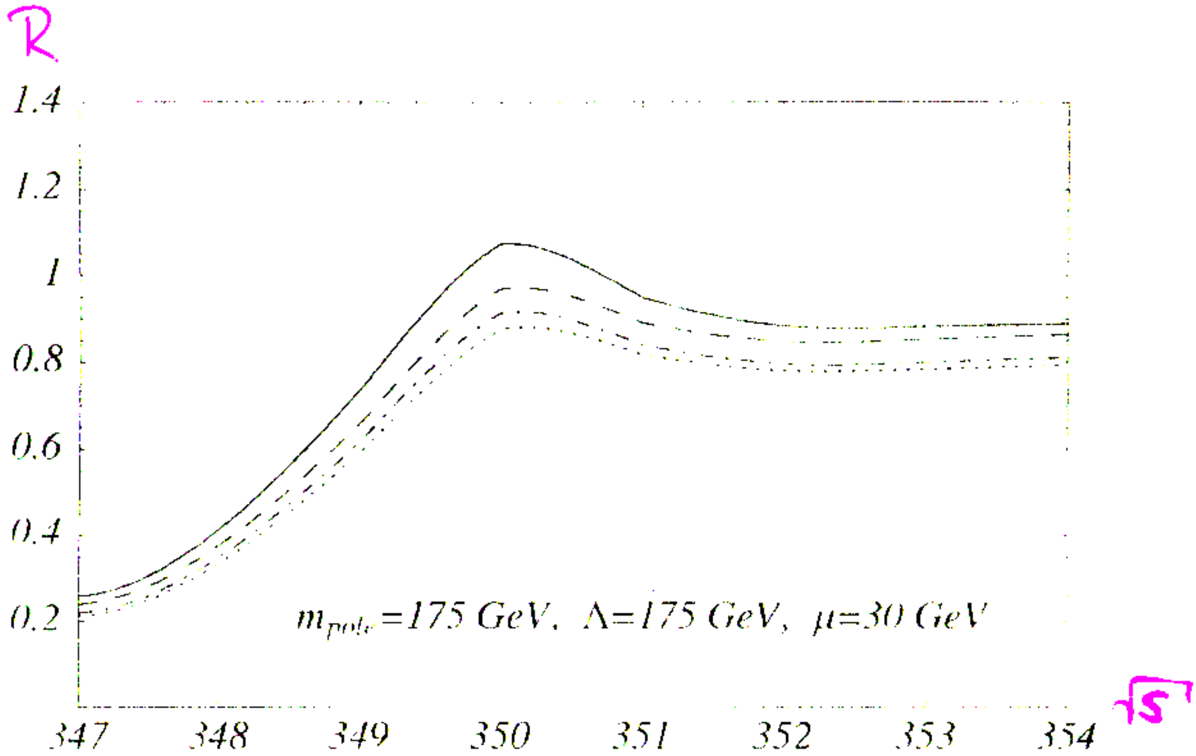
→ large NNLO correction to the normalization has nothing to do with IR sensitivity

→ Figure

⤳ DOES THIS INDICATE THE BREAKDOWN OF THE NON-RELATIVISTIC EXPANSION ?

→ Figs

Size of corrections in the normalization of σ_{tot}



········ NLO

–·–·– NLO + $O(\alpha_s^2)$ corrections in $V_{Coulomb}$

– – – – + all Abelian NNLO corrections

——— + all non-Abelian —"—
= full NNLO

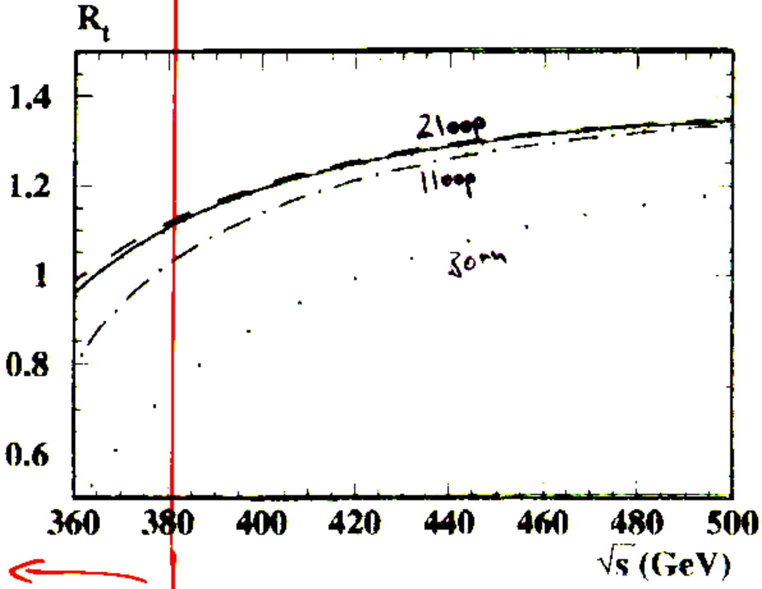
→ NNLO corrections from different sources are positive and add up

⇒ we should not be worried too much

NNLO

Size of $\mathcal{O}(\alpha_s^2)$ correction away from threshold

$\sqrt{s} > 2m_{top} + 10\text{-}15$ GeV $\Rightarrow$ perturbation theory in α_s should be valid (i.e. "converges" i.e. $\mathcal{O}(\alpha_s^3) < \mathcal{O}(\alpha_s^2)$)



(Chetyrkin, Hoang, Kühn, Steinhauser, Teubner)

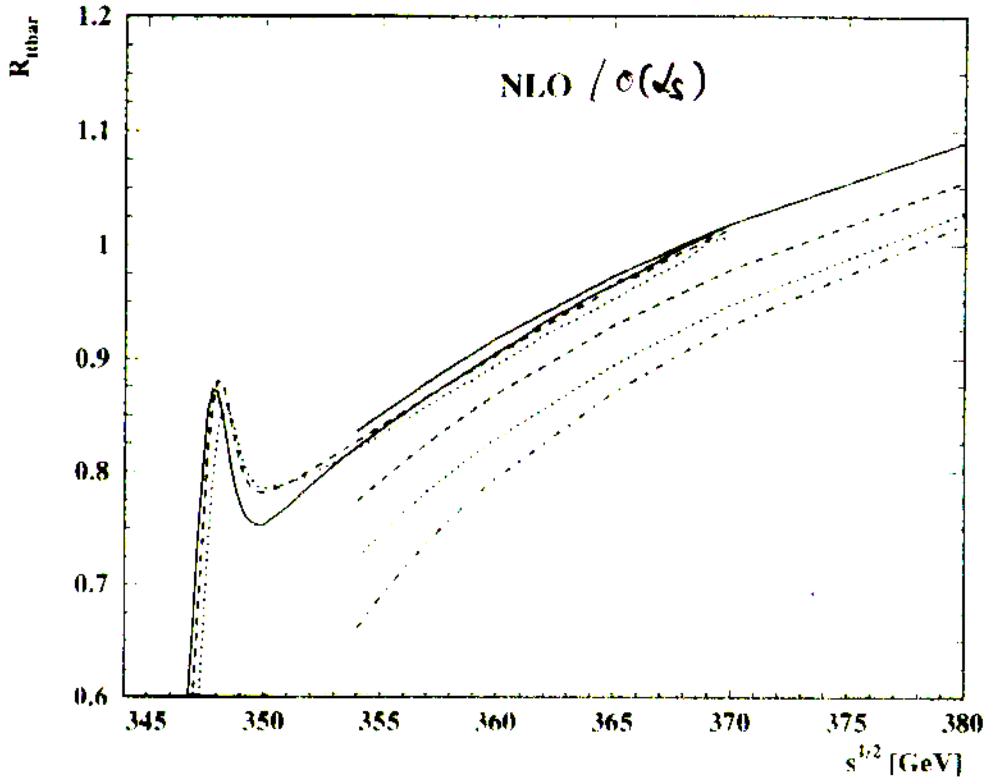
Fig. 5. The functions R_c, R_b and R_t in NLO plus dominant NNLO terms versus $\sqrt{s}$ for three different scales, $\mu^2 = M_Q^2$ (*dashed*), $\mu^2 = (2M_Q)^2$ (*solid*) and $\mu^2 = s$ (*dotted curves*). For comparison, also shown are the Born (*wide dots*) and $\mathcal{O}(\alpha_s)$ results ($\mu^2 = (2M_Q)^2$, *dash-dotted*)

$\mathcal{O}(\alpha_s^2)$ corrections are $\mathcal{O}(20\%)$ for $\sqrt{s} = 360$ GeV

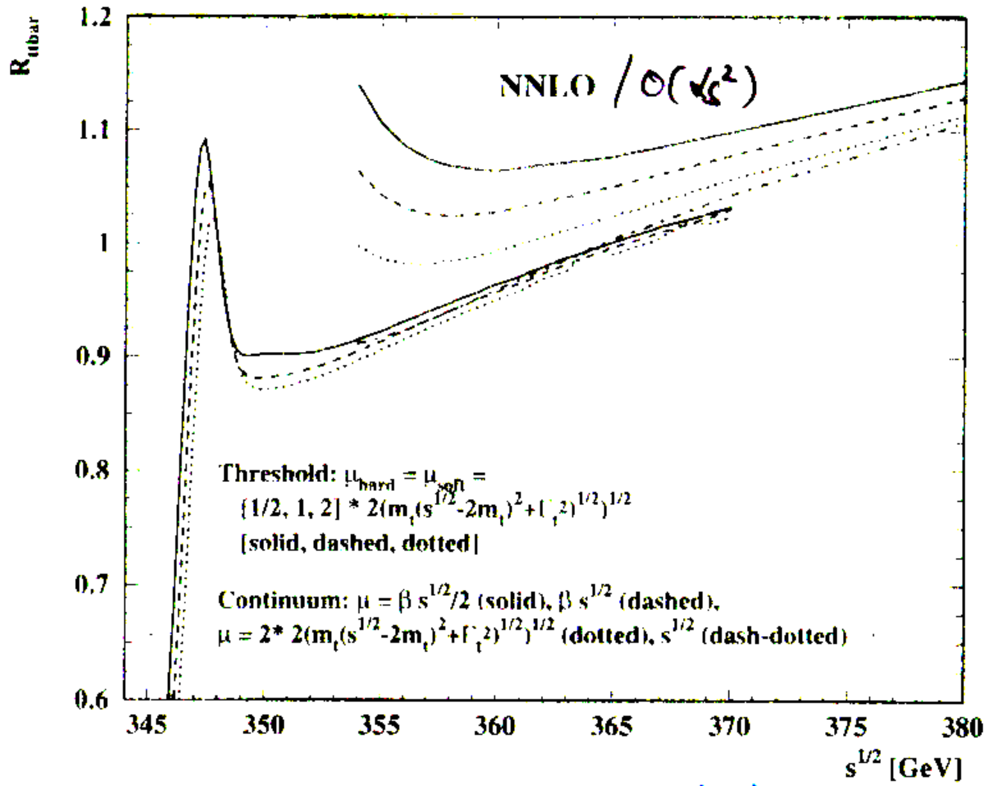
Match of $\sigma_{tot}^{\text{threshold}}$ and $\sigma_{tot}^{\text{high energy}}$

matching expected for "$\alpha_s \ll v \ll 1$" $\quad v = \sqrt{1-\frac{4m_t^2}{s}}$

$\longrightarrow \sqrt{s} \approx 365$ GeV



(Hoang, Teubner)



(Hoang, Teubner)

$\Rightarrow$ threshold resummation of $\left(\frac{\alpha_s}{v}\right)^n$ terms stabilizes the cross section

→ If we trust that pert. theory in α_s works for the intermediate energy regime, then we are most probably in the convergent regime. [That means the N^3LO corrections to the normalization are smaller than the NNLO corrections.]

$O(\alpha_s^3)$ / N^3LO $\lesssim$ 10%

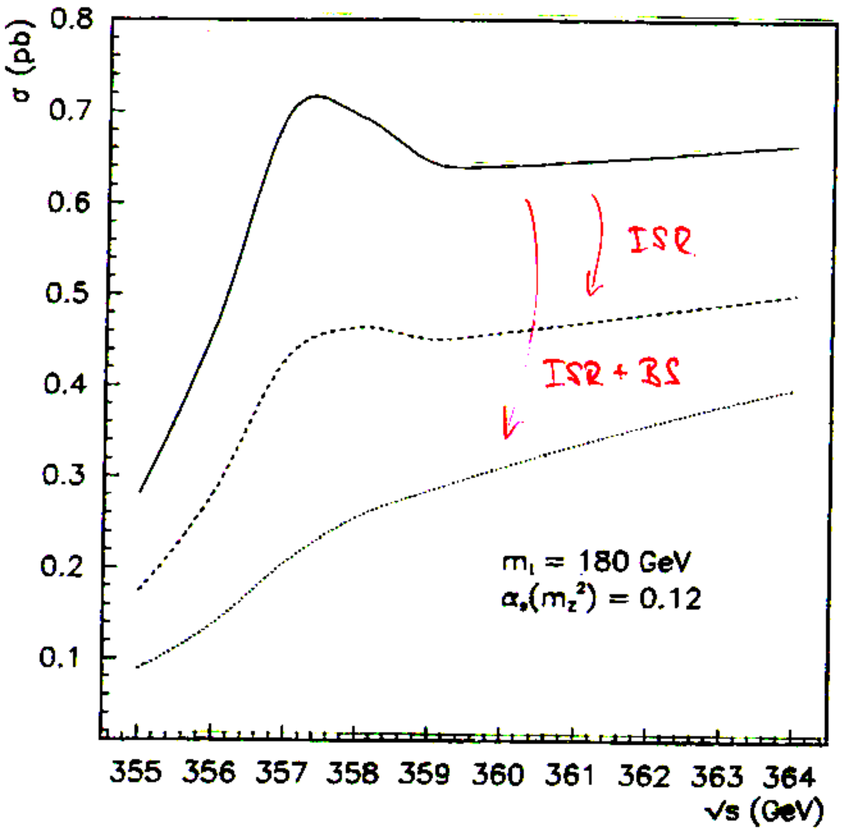
→ theoretical uncertainty in the normalization 5-10%

→ "There is nothing to worry about."

→ If the beamstrahlung effects do <u>not</u> lead to a crosstalk of uncertainties from the normalization to the top mass determination, then δM_{1S} will not be affected.

→ normalization uncertainty affects determination of α_s

Beamstrahlung (BS) smears the peak out
⇒ uncertainties in the normalization might feed
into the top mass determination



$t\bar{t}$ production cross section as a function of the centre-of-mass energy. Solid line: Born ion; dashed line: with initial state radiation (ISR); dotted line: with ISR and beam mputed with the parameters of the TESLA accelerator design [4].

Comas, Miquel, Martinez, Orteu

Summary

- EFT very efficient and powerful tool to determine $t\bar{t}$ observables close to threshold at e^+e^- linear colliders
  $\rightarrow$ NNLO calculation of σ_{tot} , NNLO corrections to $\frac{d\sigma}{d\vec{p}_t}$,
- Cutoff regularization $\longrightarrow$ exact solution of Schrödinger eq.
  $\longrightarrow$ stable results
- top mass definition matters if uncertainties of $\mathcal{O}(\Lambda_{QCD})$ or smaller are intended ; conceptually and practically pole mass <u>must be abandoned</u>.
  $\rightarrow$ 1S mass most promising $\longrightarrow$ $\delta M_{1S}^{\text{theoret.}} \lesssim 100\,\text{MeV}$
- $\overline{MS}$ mass extractable
  $\delta \overline{m}(\overline{m}) \approx 200\,\text{MeV}$ present technology
  $\delta \overline{m}(\overline{m}) \approx \delta M_{1S}$ for 3-loop $m_{pole} - \overline{m}(\overline{m})$, $\delta\alpha_s(M_Z) \approx 0.001$
- present uncertainty in the normalization of σ_{tot} : 5-10 %

Open Issues

- N^3LO corrections , retardation effects
- consistent NNLO treatment of electroweak effects
  - off-shellness , gauge invariance , background , non-factorizable , ... top decay , ...
- 3 loop relation between m_{pole} and $\overline{MS}$ mass