$$\boxed{\text{FCNC} \quad + \quad Z \to \left\{ \begin{array}{l} e + \mu \\ \mu + \tau \\ \tau + e \end{array} \right. \quad + \text{ LC at } Z\text{peak}}$$

A study started by

A. Pilaftsis, CERN TH
T. Riemann, DESY Zeuthen TH
G. Wilson, OPAL Expt., CERN

Intermediate report at LC-meeting, Oxford, March 1999

Is there a discovery potential?
- In the Standard Model (+ ν-masses)?
- In extended models?

Believe: Super-Kamiokande

$\rightsquigarrow$ $m_{Heavy} \sim 5 \times 10^{-2}$ eV

$m_{light} \sim 2 \times 10^{-3}$ eV

maximal mixing $(\tau - \mu)$

Assume: $\mathcal{L}_{LC} \sim (10 \div 100) \times \mathcal{L}_{LEP1}$

Study: $\boxed{B = \frac{\Gamma(Z \to \mu + \tau)}{\Gamma(Z \to \mu + \mu)}}$ LEP1: $B < 10^{-5}$

A look back:

$Z \to \ell_1 \bar{\ell}_2$ in Standard Model

G. Mann, T. Riemann

- Neutrino '82 (June 1982) Proc. Vol. II p. 58
- Annalen d. Physik 40 (1983) 334

} extensively used...

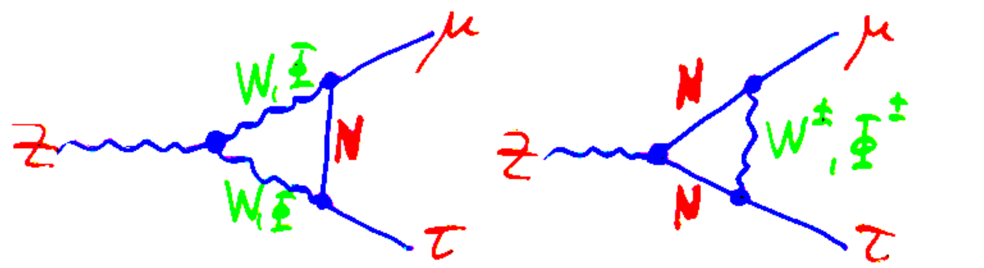
V. Ganapathi et al. (1982)

M. Clements et al. (1982)

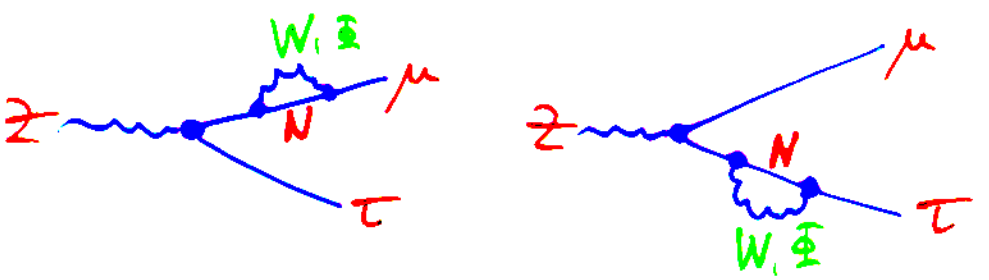
and later papers...

⋮

Pham, hep-ph/9809 322 ← simply wrong!

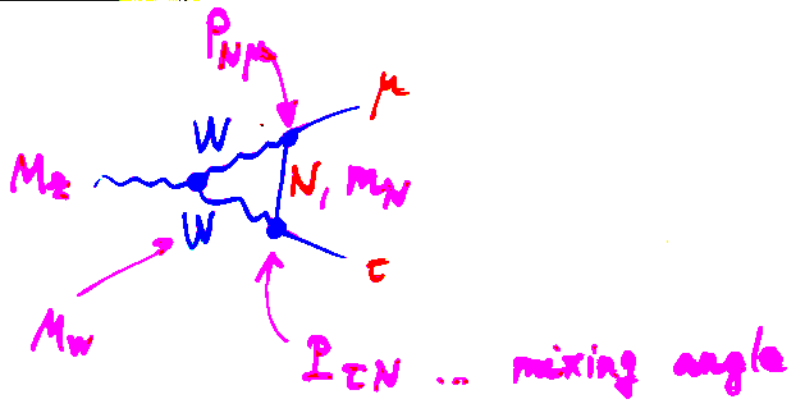


non-abelian + abelian



} counter term contributions

0x 2 TR



$g^2 \sim \alpha$

$$\boxed{\mathcal{M}^\mu \sim g^3 \sum_{i=1}^{N} P_{\mu i} \cdot P^+_{i\tau} \cdot V\left(\frac{m_i^2}{M_W^2}, \frac{M_Z^2}{M_W^2}\right) \cdot \gamma^\mu \cdot (1+\gamma_5)}$$

The matrix element vanishes if:

$V = \text{const}$: then $\sum_i P_{\mu i} P^+_{i\tau} = 0$

$P_{\mu i} = \delta_{\mu i}$: then all products $P P^+ = 0$

The branching ratio:

$$B = \frac{|\mathcal{M}|^2 \, (\mu + \bar{\tau})}{|\mathcal{M}|^2 \, (\mu + \bar{\mu})} \sim \left(\frac{\alpha}{\pi}\right)^2 \cdot \frac{1}{(4S_W^2)^2} \cdot \left|\sum_i P P^+ V(i)\right|^2$$

If one mass is bigger than all the others:

$$\underbrace{\sum_{i=1}^{N-1} P_{\mu i} \, P^+_{i\tau} \, V(0) + \boxed{P_{\mu N} \, P^+_{N\tau} \cdot V(0)}}_{=0} + P_{\mu N} \cdot P^+_{N\tau} \cdot [V(N) - V(0)]$$

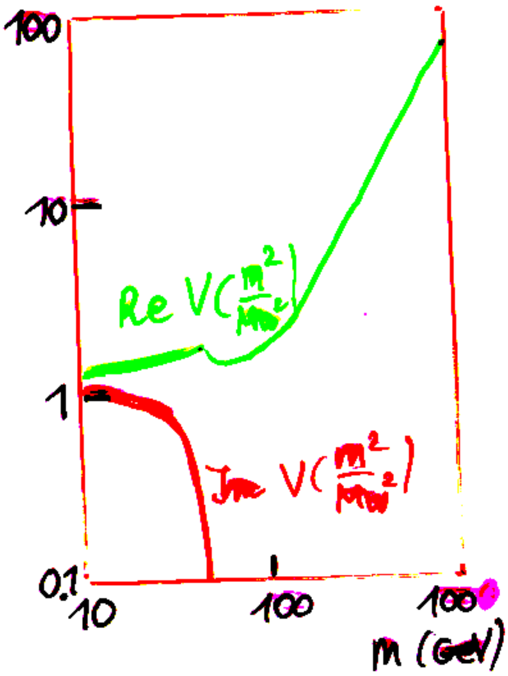
$$\boxed{B = O(1) \times \left(\frac{\alpha}{\pi}\right)^2 \cdot \frac{1}{(4S_W^2)^2} \cdot \left|P_{\mu N} \cdot P^+_{N\tau}\right|^2 \cdot \left|V\left(\frac{m_N^2}{M_W^2}\right) - V(0)\right|^2}$$

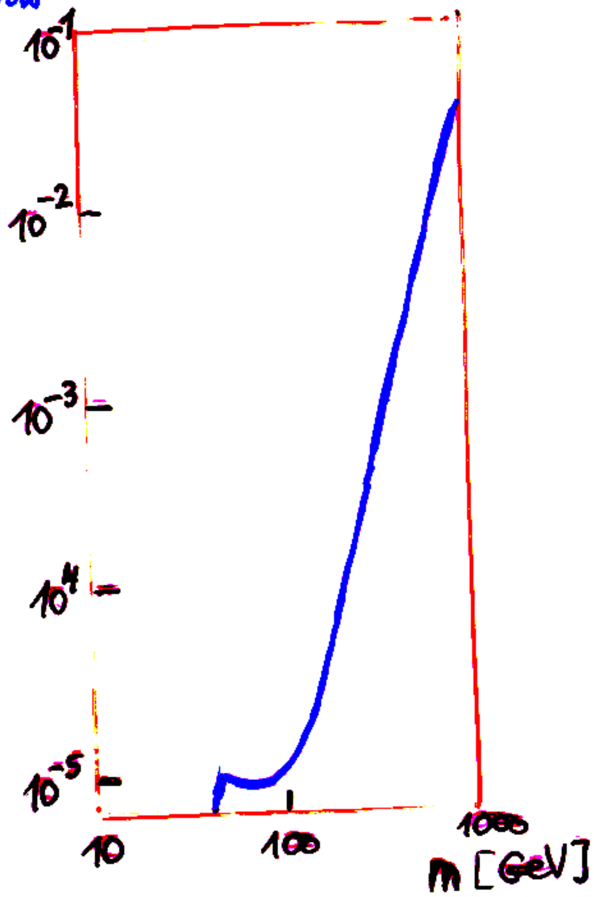
This formula holds generally, need $V\left(\frac{m^2}{M_W^2}\right)$

Ox3 TR

$$\left(\frac{\alpha}{\pi}\right)^2 \cdot \frac{1}{16 s_W^4} \left| V\left(\frac{m^2}{M_W^2}\right) - V(0) \right|^2$$

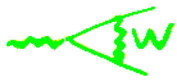
From: G. Mann, TR Ann Physik '83





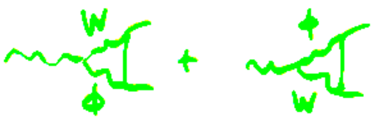
$$V(\Delta,\sigma) = \frac{1}{4}(v+a)\cdot\left[2\sigma\cdot\left(C_0 + C_{11} + C_{12} + C_{23}\right) + 4\,C_{24} - 2\right]$$

$$+\ \frac{1}{4}(v-a)\cdot 2\Delta^2\, C_0$$



$$+\ (2I_3)\cdot(-2c_w^2)\cdot\left[\sigma\cdot\left(\bar{C}_{11} + \bar{C}_{12} + \bar{C}_{23}\right) + 6\,\bar{C}_{24} - 1\right]$$

$$+\ (2I_3)\cdot(2s_w^2)\cdot\Delta\cdot\bar{C}_0$$



$$-\ (2I_3)\cdot(1-2s_w^2)\cdot\Delta\cdot\bar{C}_{24}$$

$$+\ \left\{\frac{1}{4}(v+a)\,\Delta^2 C_0 + \frac{1}{4}(v-a)\,\Delta\left[\sigma\, C_{23} + 2\cdot C_{24} - \frac{1}{2}\right]\right\}$$

$$+\ \left[\frac{1}{4}(v+a) - c_w^2\, a\right]\cdot\left[(2+\Delta)\,B_1 + 1\right]$$

C_i, B_1: Passarino, Veltman loop-fcns. $\sigma = M_Z^2/M_W^2$ and: $\Delta = \frac{m^2}{M_W^2}$

$$V(0) = (2.498 + i \times 2.057)\, \underbrace{I_3}_{=(+\frac{1}{2})} + (-0.234 - i \times 0.411)\, \underbrace{Q}_{=(0)}$$

Neutrino '82:

$$V\left(\frac{m^2}{M_W^2} \ll 1\right) = (1.25 + 1.03 \times i) + (2.53 - 2.31 \times i)\, \Delta + \dots$$

$$V\left(\frac{m^2}{M_W^2} \gg 1\right) = I_3\, (\Delta + 2.845 \ln \Delta - 4.492) + Q \cdot (-0.0486)$$

$$+ \dots$$

$$\Delta = \frac{m^2}{M_W^2}$$

$\left(\text{at: } \frac{M_Z^2}{M_W^2} = 1.25 \right)$

(1983!)

0x6

TR

change couplings and/or
particle content:
get similar numbers ...

Neutrino '82:

m_N	m_N^4/M_W^4	Branching
100 eV	2×10^{-36}	$B_Z < 2\times10^{-40}$
20 GeV	3×10^{-3}	$B_Z < 3\times10^{-7}$

$B_Z <$
since: $P_{\mu N} \cdot P_{N\tau}^{+}$

This Table demonstrates the impossibility of detecting $\boxed{Z \to \mu + \tau}$ etc.

in the Standard Model and in any model with only light neutrinos, $\boxed{m^2 \ll M_W^2}$

Conclusion:

If one would observe

$$\boxed{Z \to \mu + \tau}$$

at a rate of $B_Z \gtrsim 10^{-40}$ or so, or even $\gtrsim 10^{-7}$,
this would indicate extremely intersting new physics!

compete with other channels?
models at disposal with
e.g. $B_Z > 10^{-10}$?

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TR

Don't ask me details from now on....

Competing Reactions

$\mu \to e + \gamma$

$\mu \to e + \bar{e} + e$

$\tau \to \mu + \gamma$

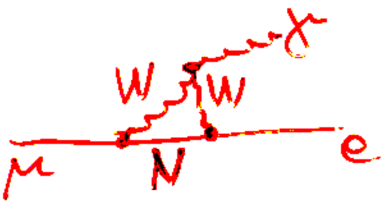
$\tau \to \mu \bar{e} e$ etc.

$\tau \to \mu \bar{\mu} e$

$\tau \to \mu \bar{\mu} \mu$, $e \bar{e} e$

$\nu \to \nu' + \gamma$

Typical diagram :



$q^2 = 0$

resembles:

$q^2 = M_Z^2$

How to make rates Big ? –

This may happen in SO(10) and similar structures....

A. Pilaftsis*, J. Bernabeu, B. Kniehl, ...

Gonzales-Garcia, Valle, ... del' Aguila ...

: * will contribute more details

<u>Interfamily seesaw</u>

$$M = \begin{pmatrix} 0 & m_D \\ m_D^+ & M_M \end{pmatrix}$$

≈ SM with right-handed neutrinos.

But : mixing suppressed

allow for left- and right-handed ν singlets :

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^+ & 0 & M^T \\ 0 & M & 0 \end{pmatrix}$$

or even :

$$\mathcal{M} = \begin{pmatrix} 0 & a & b \\ a & A & 0 \\ b & 0 & B \end{pmatrix}$$

mixing less suppressed, $M \sim O(\text{TeV})$

0x8
TA

References:

A. Pilaftsis, MPLA 9 (1994) 3595

A. Ilakovac and A. Pilaftsis, NP B437 (1995) 491

and many references cited therein ...

Ox9 TR

Conclusions

(1) Rate for $Z \to \mu\tau,\ \tau e,\ e\mu$:

$$B = \frac{\Gamma(Z \to e_1 + \bar{e}_2)}{\Gamma(Z \to e + e)} \sim \boxed{\left(\frac{\alpha}{\pi}\right)^2} \cdot \left| \sum_i \underbrace{P_{1i}\, P^+_{i2}}_{\text{potentially small}}\, V(m_i) \right|^2$$

small

$$B \sim 10^{-4} \cdot \left| V(m_N) - V(0) \right|^2 \cdot P_{1N} \cdot P^+_{N2}$$

if only one, ν_N, is heavy

(2) $B \sim \left(\frac{m_N}{M_W}\right)^4$ big? $V(m_N) \sim \frac{m_N^2}{M_W^2}$

(3) One may imagine that there are models where

$B \gtrsim 10^{-6}$ may be realized in Nature