"Renormalization Group Effects in the B.G. to Higgs production in $\gamma\gamma$ collisions"

Michael Melles
James Stirling

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Introduction

RG effects in the massive Sudakov F.F.

RG effects in the non-Sudakov F.F.

Numerical results

Summary

To calculate 1 loop DL contribution to $\gamma\gamma \to q\bar{q}$ $(J_z = 0)$, decompose 1 loop box into 'DL - phase space' Feynman diagrams:

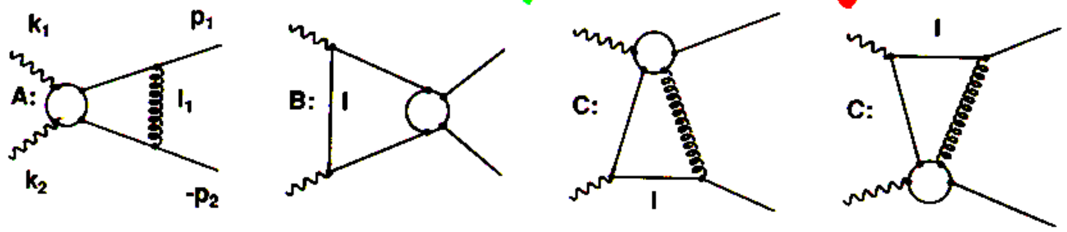


Figure 1: The schematic one loop soft (Sudakov,"A") and hard (non-Sudakov,"B,C") topologies contributing to the DL form factor. These graphs are obtained from the one loop box diagram in the process $\gamma\gamma(J_z = 0) \longrightarrow q\bar{q}$. The blob denotes a hard momentum flowing through the omitted propagator relative to the soft momentum l or l_1 in the DL phase space. For higher order DL contributions, corrections to only these topologies need to be taken into account.

→ At higher order, DL results are corrections to topologies A, B & C.

→ Explicit 3 loop DL calculation reveals factorization of Sudakov & non-Sudakov DL's:

AO:

$$\sigma^{DL}_{virt+soft} = \sigma_{Born} \left\{ 1 + \mathcal{F} \; {}_2F_2(1,1;2,\frac{3}{2};\frac{1}{2}\mathcal{F}) + 2\,\mathcal{F} \; {}_2F_2(1,1;2,\frac{3}{2};\frac{C_A}{4C_F}\mathcal{F}) \right\}^2$$
$$\exp\left(\frac{\alpha_s C_F}{\pi} \left[\log\frac{s}{m_q^2} \left(\frac{1}{2} - \log\frac{s}{4k_c^2} \right) + \log\frac{s}{4k_c^2} - 1 + \frac{\pi^2}{3} \right] \right) \qquad (1)$$

where $\mathcal{F} = -C_F \frac{\alpha_s}{4\pi} \log^2 \frac{m_q^2}{s}$ is the one loop hard form factor. The Born cross section for $(J_z = 0)$ is given by

$$\frac{d\sigma_{Born}}{d\cos\theta}(\gamma + \gamma \longrightarrow q + \bar{q}) = \frac{12\pi\alpha^2 Q_q^4}{s} \frac{\beta\,(1-\beta^4)}{(1-\beta^2\cos^2\theta)^2} \qquad (2)$$

where $\beta = \sqrt{1 - \frac{4m_q^2}{s}}$ denotes the quark velocity and Q_q the charge of the quark with mass m_q. $\alpha = \frac{1}{137}$ is the fine strucuture constant and $\sqrt{s}$ the center of mass energy of the initial photons.

But: scale of α_s is unrestricted in DL approximation!

Now: incorporate running α_s into massive Sudakov F.F.:

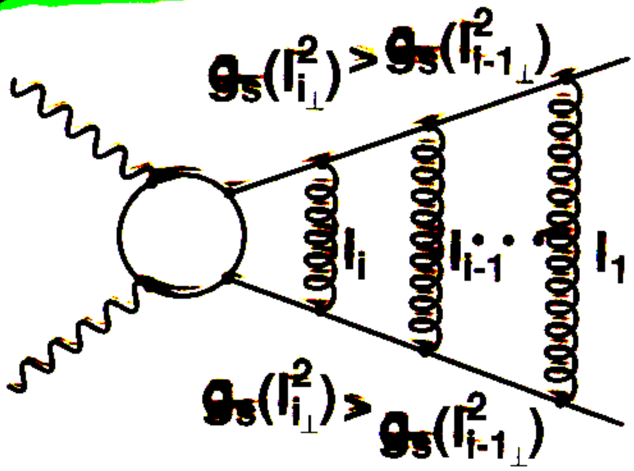


Figure 3: A schematic Feynman diagram leading to the Sudakov double logarithms in the process $\gamma\gamma(J_z = 0) \longrightarrow q\bar{q}$ with i gluon insertions. The blob denotes a hard momentum going through the omitted propagator in the DL-phase space. Crossed diagrams lead to a different ordering of the Sudakov variables with all occuring C_A terms cancelling the DL-contributions from three gluon insertions [10]. The scale of the coupling $\alpha_s = \frac{g_s^2}{4\pi}$ is indicated at the vertices and included in this work.

with the exact gauge & scheme invariant 2 loop solution of the QCD β-fct.:

$$\alpha_s(l_\perp^2) = \frac{\alpha_s(m^2)}{1 + \beta_0 \frac{\alpha_s(m^2)}{\pi} \log \frac{l_\perp^2}{m^2} + \beta_1 \left(\frac{\alpha_s(m^2)}{\pi}\right)^2 \log \frac{l_\perp^2}{m^2}} \equiv \frac{\alpha_s(m^2)}{1 + c \log \frac{l_\perp^2}{m^2}} \qquad (3)$$

In the following, we use the on-shell condition $l_\perp^2 = s\alpha\beta$

$(l \equiv \alpha \underset{\underset{q}{\uparrow}}{k_1} + \beta \underset{\underset{\bar{q}}{\uparrow}}{k_2} + l_\perp)$ (for Sudakov DL's as $\lambda \to 0$)

The DL - phase space for Sudakov logarithms:

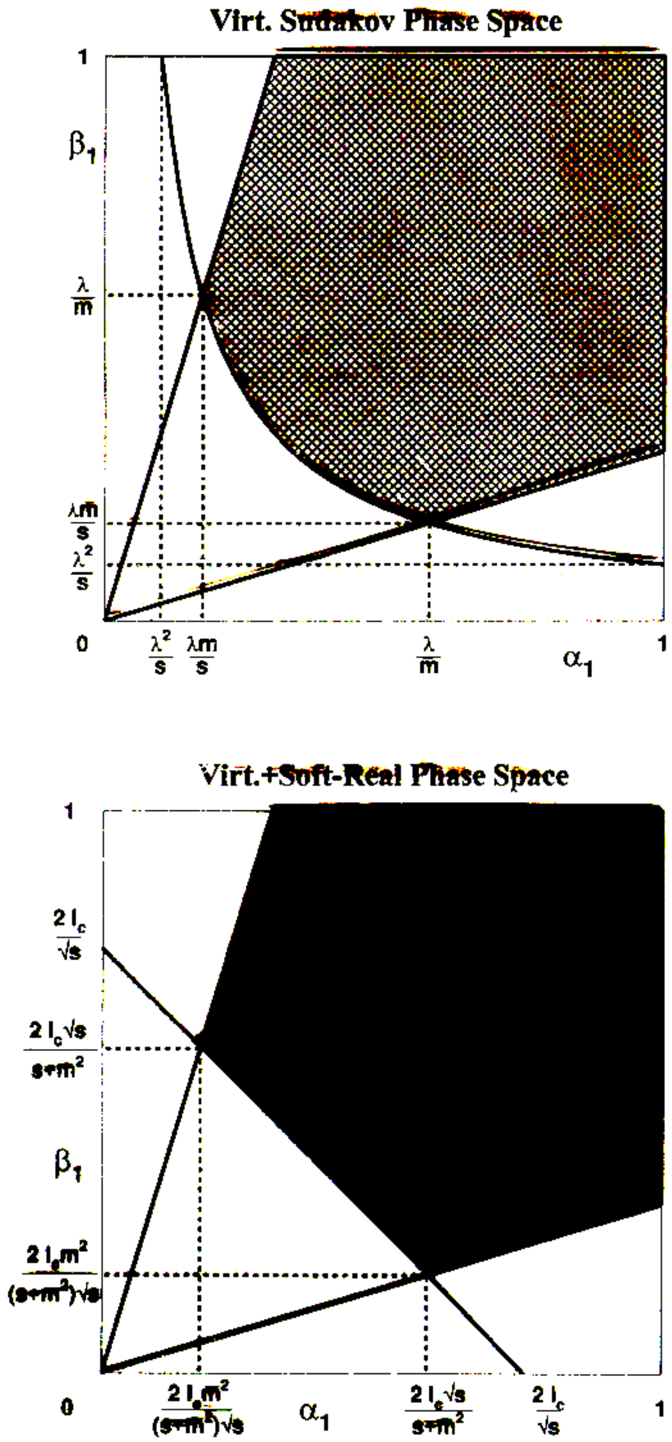


Figure 2: The schematic DL-phase space for the massive virtual (top) and the virtual plus soft real Sudakov form factor. The sum is gluon mass independent for $\lambda < \frac{2 l_c m}{\sqrt{s}}$.

The DL-result is sufficient for an RG-improvement as RG-logs occur as subleading logs at the next order.

$$
\begin{aligned}
\mathcal{F}_{S_V}^{RG} &= \overbrace{-\frac{C_F}{2\pi}\int_0^1 \frac{d\beta_1}{\beta_1}\int_0^1 \frac{d\alpha_1}{\alpha_1}\Theta(s\alpha_1\beta_1-\lambda^2)\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\Theta(\beta_1-\frac{m^2}{s}\alpha_1)}^{\text{DL phase space}} \\
&\quad \times\frac{\alpha_s(m^2)}{1+c\,\log\frac{s\alpha_1\beta_1}{m^2}} \quad \Big\}\ \text{running coupling} \\
&= -\frac{C_F}{2\pi}\left\{\int_{\frac{\lambda^2}{s}}^{\frac{\lambda}{m}}\frac{d\beta_1}{\beta_1}\int_{\frac{\lambda^2}{s\beta_1}}^{1}\frac{d\alpha_1}{\alpha_1}+\int_{\frac{\lambda}{m}}^{1}\frac{d\beta_1}{\beta_1}\int_{\frac{m^2}{s}\beta_1}^{1}\frac{d\alpha_1}{\alpha_1}\right. \\
&\quad \left.-\int_{\frac{\lambda^2}{s}}^{\frac{\lambda m}{s}}\frac{d\beta_1}{\beta_1}\int_{\frac{\lambda^2}{s\beta_1}}^{1}\frac{d\alpha_1}{\alpha_1}-\int_{\frac{\lambda m}{s}}^{\frac{m^2}{s}}\frac{d\beta_1}{\beta_1}\int_{\frac{\lambda}{m^2}\beta_1}^{1}\frac{d\alpha_1}{\alpha_1}\right\}\frac{\alpha_s(m^2)}{1+c\,\log\frac{s\alpha_1\beta_1}{m^2}} \\
&= -\frac{\alpha_s(m^2)C_F}{2\pi}\left\{\frac{1}{c}\log\frac{s}{m^2}\left(\log\frac{\alpha_s(\lambda^2)}{\alpha_s(s)}-1\right)+\frac{1}{c^2}\log\frac{\alpha_s(m^2)}{\alpha_s(s)}\right\} \qquad (17)
\end{aligned}
$$

→ Exact RG-improved result for the massive virtual Sudakov F.F.

Does it exponentiate? Check 2 loops explicitly!

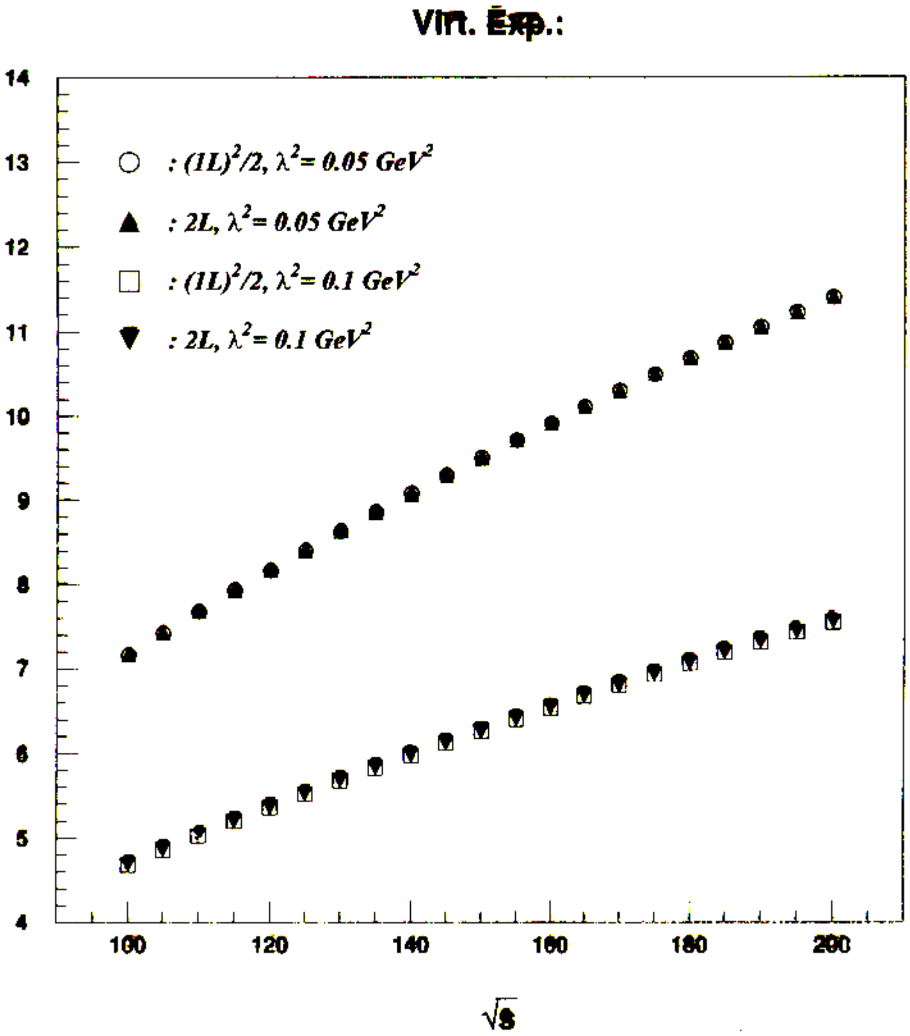


Figure 8: A comparison of the second term in the exponential obtained from the renormalization group improved massive one-loop Sudakov form factor compared with the explicit two loop results. The quark mass is kept fixed at $m_b = 4.5$ GeV and the value for the gluon mass λ is indicated in the figure. The result in Eq. 17 clearly exponentiates.

possible only numerically (VEGAS):

$$
\begin{aligned}
\mathcal{F}_{S_V}^{RG_{2L}} &= \frac{C_F^2}{4\pi^2}\int_0^1 \frac{d\beta_1}{\beta_1}\int_0^1 \frac{d\alpha_1}{\alpha_1}\int_0^1 \frac{d\beta_2}{\beta_2}\int_0^1 \frac{d\alpha_2}{\alpha_2}\,\frac{\alpha_s(m^2)}{1+c\,\log\frac{s\alpha_1\beta_1}{m^2}}\,\frac{\alpha_s(m^2)}{1+c\,\log\frac{s\alpha_2\beta_2}{m^2}} \\
&\times \Big\{\Theta(s\alpha_1\beta_1-\lambda^2)\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\Theta(\beta_1-\frac{m^2}{s}\alpha_1)\Theta(s\alpha_2\beta_2-\lambda^2) \\
&\quad \Theta(\alpha_2-\frac{m^2}{s}\beta_2)\Theta(\beta_2-\frac{m^2}{s}\alpha_2)\Theta(\alpha_2-\alpha_1)\Theta(\beta_2-\beta_1) \;+ \\
&\quad \Theta(s\alpha_1\beta_1-\lambda^2)\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\Theta(\beta_1-\frac{m^2}{s}\alpha_2)\Theta(s\alpha_2\beta_2-\lambda^2) \\
&\quad \Theta(\alpha_2-\frac{m^2}{s}\beta_1)\Theta(\beta_2-\frac{m^2}{s}\alpha_2)\Theta(\alpha_2-\alpha_1)\Theta(\beta_1-\beta_2)\Big\}
\end{aligned}
\qquad (18)
$$

→ Combine with 1 loop subleading terms, now, however, scale-fixed!

$$
\begin{aligned}
\mathcal{F}_{S_V}^{RG} = & -\frac{\alpha_s(m^2)C_F}{2\pi}\left\{\frac{1}{c}\log\frac{s}{m^2}\left(\log\frac{\alpha_s(\lambda^2)}{\alpha_s(s)}-1\right)+\frac{1}{c^2}\log\frac{\alpha_s(m^2)}{\alpha_s(s)}\right. \\
& \left. -\frac{1}{2}\log\frac{s}{m^2}-\log\frac{m^2}{\lambda^2}+1-\frac{2\pi^2}{3}\right\} \qquad (19)
\end{aligned}
$$

$\mathcal{F}_{S_V}^{RG}$ exponentiates!

λ- dependence must cancel with real Bremsstrahlung contribution!

For Real Soft contribution, introduce energy cut: $\lambda \leq l_0 \leq l_c$?:

The DL result for the virtual one loop Sudakov form factor can be written in terms of integrals over α_1 and β_1 as:

$$
\begin{aligned}
\mathcal{F}^{DL}_{S_V} &= -\frac{\alpha_s C_F}{2\pi}\int_0^1 \frac{d\beta_1}{\beta_1}\int_0^1 \frac{d\alpha_1}{\alpha_1}\Theta(s\alpha_1\beta_1-\lambda^2)\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\Theta(\beta_1-\frac{m^2}{s}\alpha_1) \\
&= -\frac{\alpha_s C_F}{2\pi}\left(\frac{1}{2}\log^2\frac{s}{m^2}+\log\frac{s}{m^2}\log\frac{m^2}{\lambda^2}\right) \qquad (4)
\end{aligned}
$$

For the inclusion of double logarithms from the real Bremsstrahlung contribution we introduce a cutoff for the energy integration according to $\lambda \leq l_0 \leq l_c$, i.e. in terms of Sudakov variables, $\alpha_1+\beta_1 \leq \frac{l_c}{E_{cm}} = \frac{2l_c}{\sqrt{s}}$. Thus we find for the real DL Bremsstrahlung contribution:

$$
\begin{aligned}
\mathcal{F}^{DL}_{S_R} &= \frac{\alpha_s C_F}{\pi}\int_0^1 \frac{d\beta_1}{\beta_1}\int_0^1 \frac{d\alpha_1}{\alpha_1}\Theta(s\alpha_1\beta_1-\lambda^2)\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\Theta(\beta_1-\frac{m^2}{s}\alpha_1) \\
&\quad\times\Theta\left(\frac{2l_c}{\sqrt{s}}-\alpha_1-\beta_1\right) \\
&= -2\mathcal{F}^{DL}_{S_V} - \frac{\alpha_s C_F}{\pi}\int_0^1 \frac{d\beta_1}{\beta_1}\int_0^1 \frac{d\alpha_1}{\alpha_1}\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\Theta(\beta_1-\frac{m^2}{s}\alpha_1) \\
&\quad\times\Theta\left(\alpha_1+\beta_1-\frac{2l_c}{\sqrt{s}}\right) \\
&= \frac{\alpha_s C_F}{\pi}\left(\frac{1}{2}\log^2\frac{s}{m^2}+\log\frac{s}{m^2}\log\frac{m^2}{\lambda^2}-\log\frac{s}{m^2}\log\frac{s}{4l_c^2}\right) \qquad (5)
\end{aligned}
$$

assuming only $\lambda < \frac{2l_c m}{\sqrt{s}}$.

Now, with a running coupling $\alpha_s(l_\perp^2)$:

$$
\begin{aligned}
\mathcal{F}^{RG}_{S_R}+2\mathcal{F}^{RG}_{S_V} &= -\frac{C_F}{\pi}\left\{\int_{\frac{2l_c m^2}{(s+m^2)\sqrt{s}}}^{\frac{2l_c\sqrt{s}}{s+m^2}}\frac{d\beta_1}{\beta_1}\int_{\frac{2l_c}{\sqrt{s}}-\beta_1}^1\frac{d\alpha_1}{\alpha_1}+\int_{\frac{2l_c}{\sqrt{s}}}^1\frac{d\beta_1}{\beta_1}\int_{\frac{m^2}{s}\beta_1}^1\frac{d\alpha_1}{\alpha_1}\right. \\
&\quad\left.-\int_{\frac{2l_c m^2}{s\sqrt{s}}}^{\frac{m^2}{s}}\frac{d\beta_1}{\beta_1}\int_{\frac{s}{m^2}\beta_1}^1\frac{d\alpha_1}{\alpha_1}\right\}\frac{\alpha_s(m^2)}{1+c\,\log\frac{s\alpha_1\beta_1}{m^2}} \\
&= -\frac{\alpha_s(m^2)C_F}{\pi\,c}\left\{-\int_{\frac{2l_c m^2}{(s+m^2)\sqrt{s}}}^{\frac{2l_c\sqrt{s}}{s+m^2}}\frac{d\beta_1}{\beta_1}\log\frac{1+c\,\log\left(\left(\frac{2l_c}{\sqrt{s}}-\beta_1\right)\beta_1\frac{s}{m^2}\right)}{\left(1+c\,\log\frac{s\beta_1}{m^2}\right)}\right. \\
&\quad+\log\frac{s}{m^2}\log\frac{\alpha_s(2l_c\sqrt{s})}{\alpha_s(s)}+\log\frac{2l_c}{\sqrt{s}}\log\frac{\alpha_s(2l_c\sqrt{s})}{\alpha_s\left(\frac{2l_c m^2}{\sqrt{s}}\right)} \\
&\quad\left.+\frac{1}{c}\log\frac{\alpha_s(m^2)\alpha_s(2l_c\sqrt{s})}{\alpha_s(s)\alpha_s\left(\frac{2l_c m^2}{\sqrt{s}}\right)}\right\} \qquad (6)
\end{aligned}
$$

Does the sum exponentiate? Check at 2 loops:

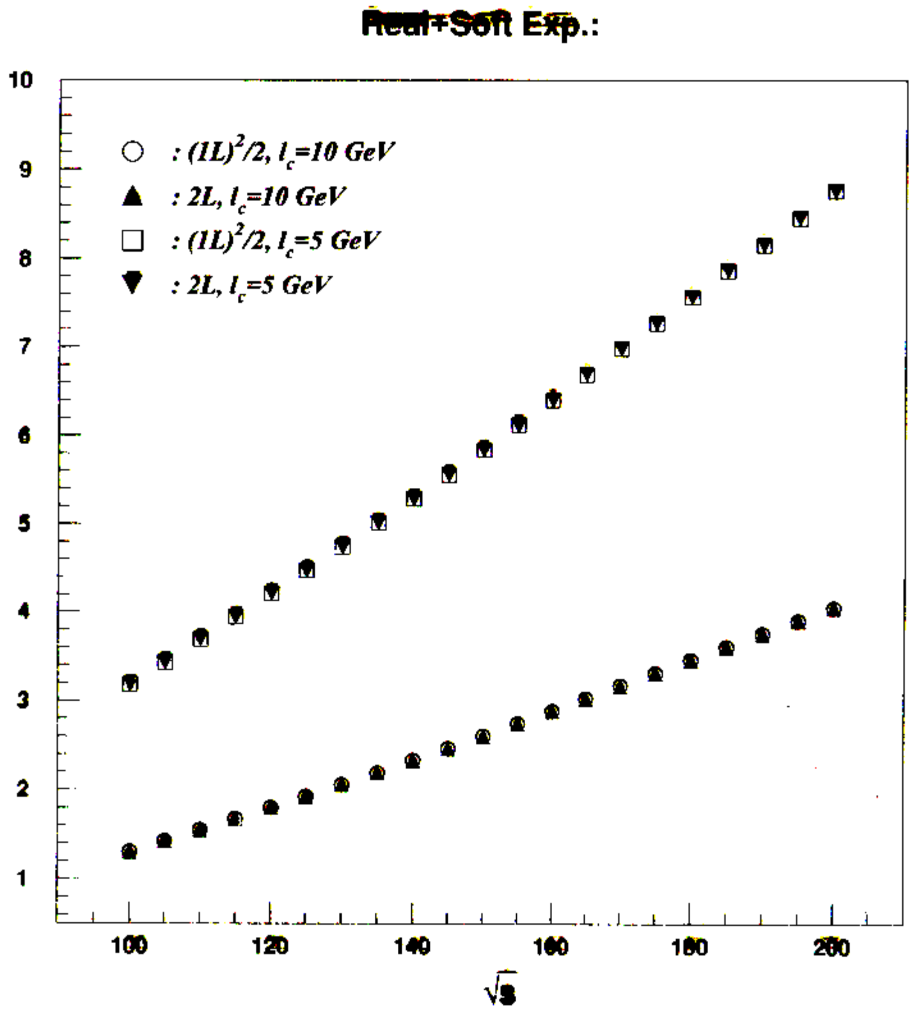


Figure 4: A comparison of the second term in the exponential obtained from the renormalization group improved massive one-loop Sudakov form factor compared with the explicit two loop results. The quark mass is kept fixed at $m_b = 4.5$ GeV and the value for the real gluon energy cutoff l_c is indicated in the figure. The result in Eq. 6 clearly exponentiates.

(use VEGAS :)

$$\begin{aligned}
\mathcal{F}^{DL_{2L}}_{S_R} + 2\mathcal{F}^{DL_{2L}}_{S_V} &= \frac{C_F^2}{4\pi^2}\int_0^1 \frac{d\beta_1}{\beta_1}\int_0^1 \frac{d\alpha_1}{\alpha_1}\int_0^1 \frac{d\beta_2}{\beta_2}\int_0^1 \frac{d\alpha_2}{\alpha_2}\, \frac{\alpha_s(m^2)}{1+c\,\log\frac{s\alpha_1\beta_1}{m^2}}\, \frac{\alpha_s(m^2)}{1+c\,\log\frac{s\alpha_2\beta_2}{m^2}} \\
&\times\ \Theta\left(\alpha_1+\beta_1-\frac{2l_c}{\sqrt{s}}\right)\Theta\left(\alpha_2+\beta_2-\frac{2l_c}{\sqrt{s}}\right)\left\{\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\right. \\
&\quad \Theta(\beta_1-\frac{m^2}{s}\alpha_1)\Theta(\alpha_2-\alpha_1)\Theta(\alpha_2-\frac{m^2}{s}\beta_2)\Theta(\beta_2-\frac{m^2}{s}\alpha_2) \\
&\quad \Theta(\beta_2-\beta_1)+\Theta(\alpha_1-\frac{m^2}{s}\beta_1)\Theta(\beta_1-\frac{m^2}{s}\alpha_2)\Theta(\alpha_2-\alpha_1) \\
&\quad \left.\Theta(\alpha_2-\frac{m^2}{s}\beta_1)\Theta(\beta_2-\frac{m^2}{s}\alpha_2)\Theta(\beta_1-\beta_2)\right\}
\end{aligned} \tag{20}$$

Again we must add the 1 loop subleading terms,

now scale-fixed:

$$\mathcal{F}^{RG}_{S_R} + 2\mathcal{F}^{RG}_{S_V} = \frac{\alpha_s(m^2)C_F}{\pi}\left\{\frac{1}{c}\int_{\frac{2l_c m^2}{(s+m^2)\sqrt{s}}}^{\frac{2l_c\sqrt{s}}{s+m^2}} \frac{d\beta_1}{\beta_1}\log\frac{1+c\,\log\left(\left(\frac{2l_c}{\sqrt{s}}-\beta_1\right)\beta_1\frac{s}{m^2}\right)}{\left(1+c\,\log\frac{s\beta_1}{m^2}\right)} \right.$$
$$-\frac{1}{c}\log\frac{s}{m^2}\log\frac{\alpha_s(2l_c\sqrt{s})}{\alpha_s(s)} - \frac{1}{c}\log\frac{2l_c}{\sqrt{s}}\log\frac{\alpha_s(2l_c\sqrt{s})}{\alpha_s\left(\frac{2l_c m^2}{\sqrt{s}}\right)}$$
$$\left. -\frac{1}{c^2}\log\frac{\alpha_s(m^2)\alpha_s(2l_c\sqrt{s})}{\alpha_s(s)\alpha_s\left(\frac{2l_c m^2}{\sqrt{s}}\right)} + \frac{1}{2}\log\frac{s}{m^2} + \log\frac{s}{4l_c^2} - 1 + \frac{\pi^2}{3}\right\} \quad (7)$$

assuming only $\frac{m^2}{s} \ll 1$.

$\left(\mathcal{F}^{RG}_{S_R} + 2\,\mathcal{F}^{RG}_{S_V}\right)$ exponentiates.

The l_c-dependence must cancel order by order with the inclusion of the hard Bremsstrahlung contributions.

The non-Sudakov topologies:

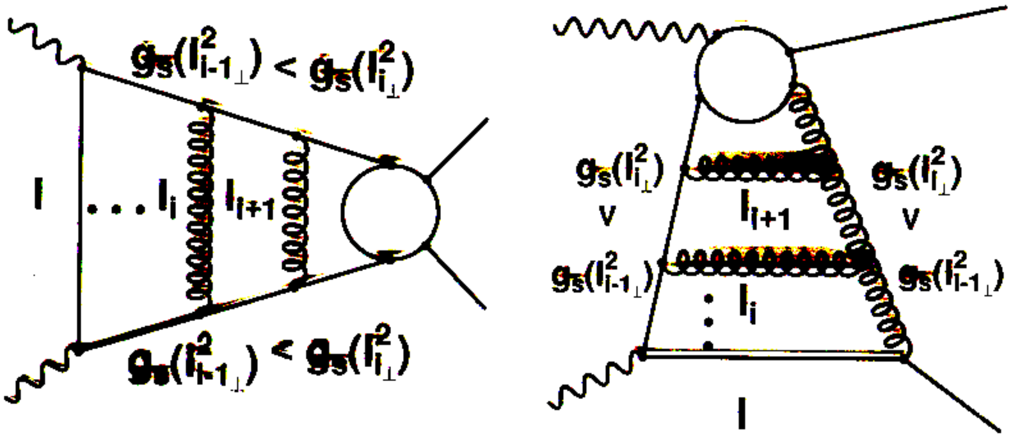


Figure 5: The schematic Feynman diagrams leading to the hard (non-Sudakov) double logarithms in the process $\gamma\gamma(J_z = 0) \longrightarrow q\bar{q}$ with $i+1$ gluon insertions. The blobs denote a hard momentum going through the omitted propagator in the DL-phase space. Crossed diagrams lead to a different ordering of the Sudakov variables and are correctly accounted for by a factor of $(i+1)!$ at each order. The scale of the coupling $\alpha_s = \frac{g_s^2}{4\pi}$ is indicated at the vertices and included in this work. The topology on the left side is Abelian like, the one on the right non-Abelian beyond one loop.

They contain no soft DL's.

$\rightarrow$ include running coupling $\alpha_s(\mu^2)$

$\mu^2 = \vec{l}_\perp^{\,2}$ as in Sudakov case:

(same topology but regularized by fermion mass;
in DL phase space, $\frac{m^2}{s} \ll \vec{l}^{\,2}_\perp/s \ll 1$)

α_s is not renormalized by last loop integration over fermion line!

→ reformulate ${}_2F_2$ - series from $\{\alpha, \beta\}$ into $\{\alpha, \vec{l}_\perp^{\,2}\}$:

these contributions were derived by first integrating over all $l_\perp^2$ with integrals over $\{\alpha, \beta\}$ pairs remaining. For the topology depicted on the left in Fig. 5 we find that a simple reformulation leads to:

$$
\begin{aligned}
\mathcal{F}\;{}_2F_2(1,1;2,\tfrac{3}{2};\tfrac{1}{2}\mathcal{F}) &= \sum_{n=1}^{\infty}\int_{\frac{m^2}{s}}^{1}\frac{d\alpha}{\alpha}\int_{\frac{m^2}{s\alpha}}^{1}\frac{d\beta}{\beta}\prod_{i=0}^{n-2}\Gamma(n)\int_{\alpha_i}^{1}\frac{d\alpha_{i+1}}{\alpha_{i+1}}\int_{\beta_i}^{1}\frac{d\beta_{i+1}}{\beta_{i+1}}\left(\frac{-\alpha_s C_F}{2\pi}\right)^n \\
&= \sum_{n=1}^{\infty}\int_{m^2}^{s}\frac{d\mathbf{l}_\perp^2}{\mathbf{l}_\perp^2}\int_{\frac{\mathbf{l}_\perp^2}{s}}^{1}\frac{d\alpha}{\alpha}\prod_{i=0}^{n-2}\Gamma(n)\int_{m^2}^{\mathbf{l}_{i_\perp}^2}\frac{d\mathbf{l}_{i+1_\perp}^2}{\mathbf{l}_{i+1_\perp}^2}\int_{\alpha_i}^{1}\frac{d\alpha_{i+1}}{\alpha_{i+1}}\left(\frac{-\alpha_s C_F}{2\pi}\right)^n \quad (8)
\end{aligned}
$$

where $\mathcal{F} = -\frac{\alpha_s C_F}{4\pi}\log^2\frac{s}{m^2}$ denotes the hard one loop form factor of Ref. [10]. The product above is set to one for $n = 1$ and contains nested integrals for $n \geq 2$ with $\mathbf{l}_{0_\perp}^2 \equiv \mathbf{l}_\perp^2$ and $\alpha_0 \equiv \alpha$. From this expression it is clear that an incorporation of the running coupling in Eq. 3 will not contain any Landau pole singularity [18] as $m^2 \leq \mathbf{l}_{i_\perp}^2 \leq \mathbf{l}_\perp^2$. We now include the running of α_s according to Eq. 3 as follows. For each gluon insertion we have

no Landau pole!

$$
\begin{aligned}
\int_{m^2}^{\mathbf{l}_{i_\perp}^2}\frac{d\mathbf{l}_{i+1_\perp}^2}{\mathbf{l}_{i+1_\perp}^2}\int_{\alpha_i}^{1}\frac{d\alpha_{i+1}}{\alpha_{i+1}}\frac{\alpha_s(m^2)}{1+c\,\log\frac{\mathbf{l}_{i+1_\perp}^2}{m^2}} &= -\frac{\alpha_s(m^2)}{c}\log\alpha_i\log(1+c\,\log\frac{\mathbf{l}_{i_\perp}^2}{m^2}) \\
&= -\frac{\alpha_s(m^2)}{c}\log\alpha_i\log\frac{\alpha(m^2)}{\alpha(\mathbf{l}_{i_\perp}^2)} \qquad (9)
\end{aligned}
$$

with

$$
\frac{d\mathbf{l}_{i_\perp}^2}{\mathbf{l}_{i_\perp}^2} = -\frac{\alpha_s(m^2)}{c}\frac{d\log(\alpha_s(\mathbf{l}_{i_\perp}^2))}{\alpha_s(\mathbf{l}_{i_\perp}^2)} \qquad (10)
$$

we find

$$
\int_{m^2}^{\mathbf{l}_{i-1_\perp}^2}\frac{d\mathbf{l}_{i_\perp}^2}{\mathbf{l}_{i_\perp}^2}\alpha_s(\mathbf{l}_{i_\perp}^2)\left(-\frac{\alpha_s(m^2)}{c}\log\frac{\alpha(m^2)}{\alpha(\mathbf{l}_{i_\perp}^2)}\right) = \frac{\alpha_s^2(m^2)}{2\,c^2}\log^2\frac{\alpha(m^2)}{\alpha(\mathbf{l}_{i-1_\perp}^2)} \qquad (11)
$$

It is clear from this derivation that for i-gluon iterations we have

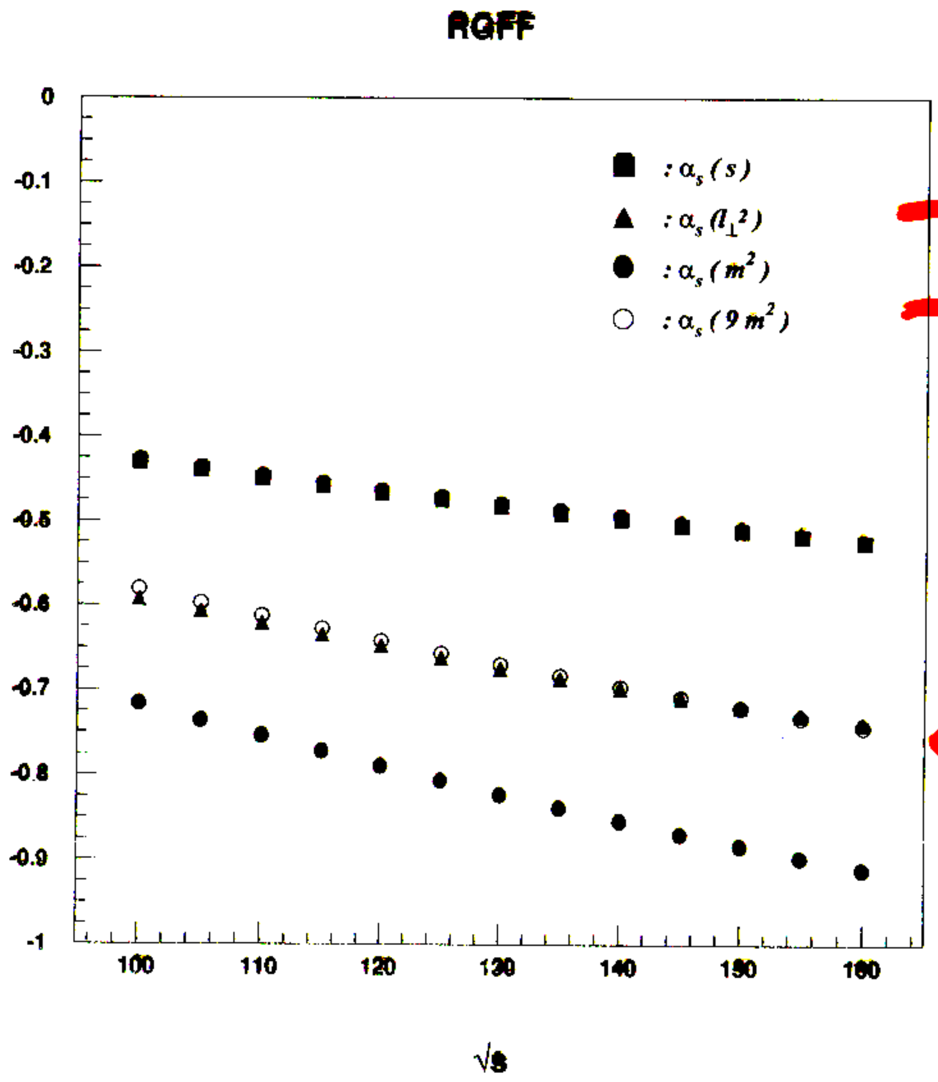
$$
\frac{(-1)^{i+1}}{i!}\left(\frac{\alpha_s(m^2)}{c}\right)^i\frac{\log^i\alpha}{\alpha}\log^i\frac{\alpha_s(\mathbf{l}_\perp^2)}{\alpha_s(m^2)} \qquad (12)
$$

Thus we finally arrive at the complete renormalization group improved result for the hard non-Sudakov form factor corresponding to the left ("Abelian") topology in Fig. 5:

$$
\begin{aligned}
\mathcal{F}_{ARG}^{h} &= \sum_{i=0}^{\infty}\int_{m^2}^{s}\frac{d\mathbf{l}_\perp^2}{\mathbf{l}_\perp^2}\int_{\frac{\mathbf{l}_\perp^2}{s}}^{1}\frac{d\alpha}{\alpha}\left(\frac{-C_F}{2\pi}\right)^{i+1}\left(\frac{\alpha_s(m^2)}{c}\right)^i\frac{\alpha_s(\mathbf{l}_\perp^2)}{i!}\frac{\log^i\alpha}{\alpha}\log^i\frac{\alpha_s(\mathbf{l}_\perp^2)}{\alpha_s(m^2)} \\
&= \sum_{i=0}^{\infty}\int_{m^2}^{s}\frac{d\mathbf{l}_\perp^2}{\mathbf{l}_\perp^2}\left(\frac{C_F}{2\pi}\right)^{i+1}\left(\frac{-\alpha_s(m^2)}{c}\right)^i\frac{\alpha_s(\mathbf{l}_\perp^2)}{(i+1)!}\log^{i+1}\frac{\mathbf{l}_\perp^2}{s}\log^i\frac{\alpha_s(\mathbf{l}_\perp^2)}{\alpha_s(m^2)} \qquad (13)
\end{aligned}
$$

Abelian RG FF (includes exact gauge & scheme invariant running coupling)

Purely hard FF:



effective α-scale ~ $9m^2$

Figure 6: The effect of incorporating a running coupling constant at each loop integration according to Eq. 13. The values of the form factor in the DL approximation of Ref. [10] are also shown with their indicated scale of where α_s was evaluated. If one chooses an effective scale of roughly $9m^2$ one obtains results very close to the exact renormalization group improved values.

$$C_F^{i+1} \longrightarrow C_F \left(\frac{C_A}{2}\right)^i \tag{14}$$

and a factor 2 as this topology occurs twice in the process $\gamma\gamma \longrightarrow q\bar{q}$. In summary, the complete virtual renormalization group improved hard form factor is thus given by

$$\begin{aligned} \mathcal{F}^h_{RG} = & \sum_{i=0}^{\infty} \int_{m^2}^{s} \frac{dl_\perp^2}{l_\perp^2} \left(\frac{C_F}{2\pi}\right)^{i+1} \left(\frac{\alpha_s(m^2)}{c}\right)^i \frac{\alpha_s(l_\perp^2)}{(i+1)!} \log^{i+1}\frac{l_\perp^2}{s} \log^i \frac{\alpha_s(m^2)}{\alpha_s(l_\perp^2)} + \\ & 2\sum_{i=0}^{\infty} \int_{m^2}^{s} \frac{dl_\perp^2}{l_\perp^2} \frac{C_F C_A^i}{2^{2i+1}\pi^{i+1}} \left(\frac{\alpha_s(m^2)}{c}\right)^i \frac{\alpha_s(l_\perp^2)}{(i+1)!} \log^{i+1}\frac{l_\perp^2}{s} \log^i \frac{\alpha_s(m^2)}{\alpha_s(l_\perp^2)} \end{aligned} \tag{15}$$

and thus

$$\frac{\sigma^{DL}_{RG}}{\sigma_{Born}} = \left\{1 + \mathcal{F}^h_{RG}\right\}^2 \exp\left(\mathcal{F}^{RG}_{S_R} + 2\mathcal{F}^{RG}_{S_V}\right) \tag{16}$$

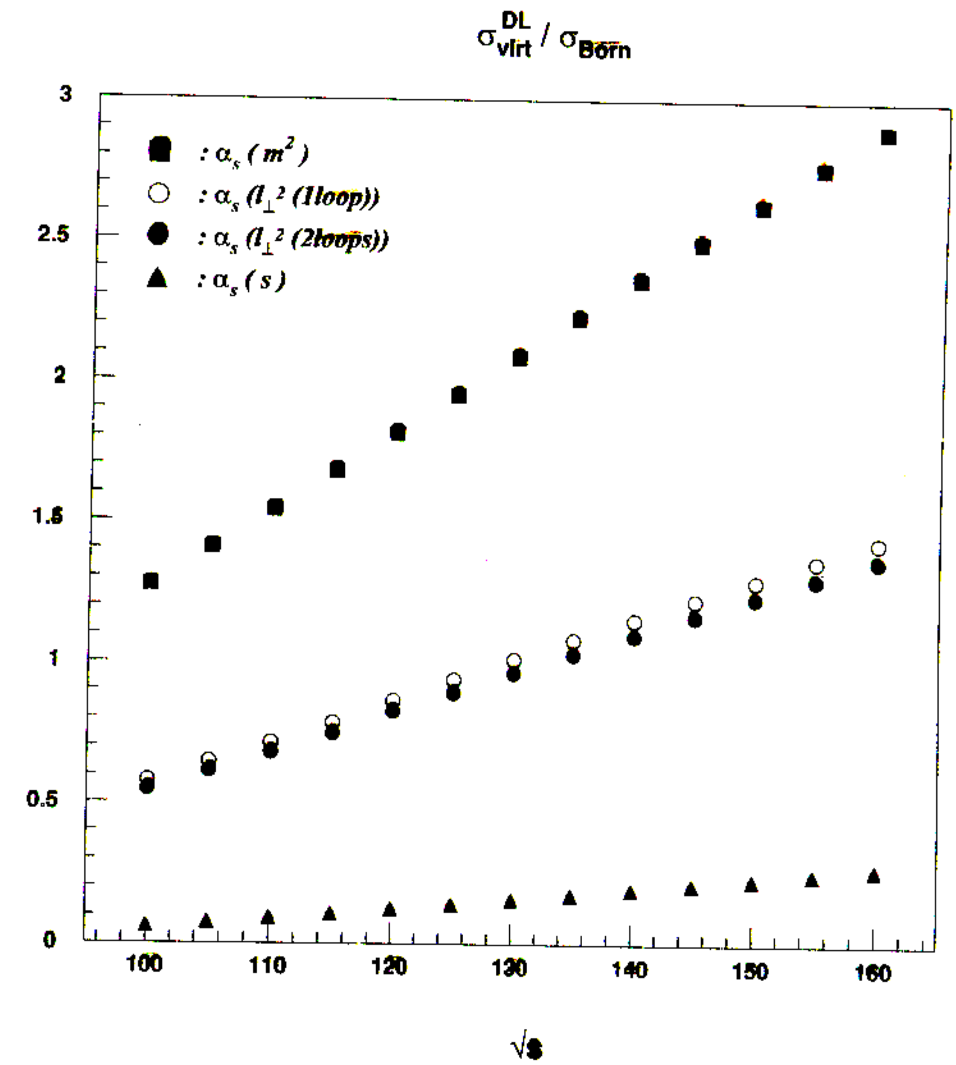
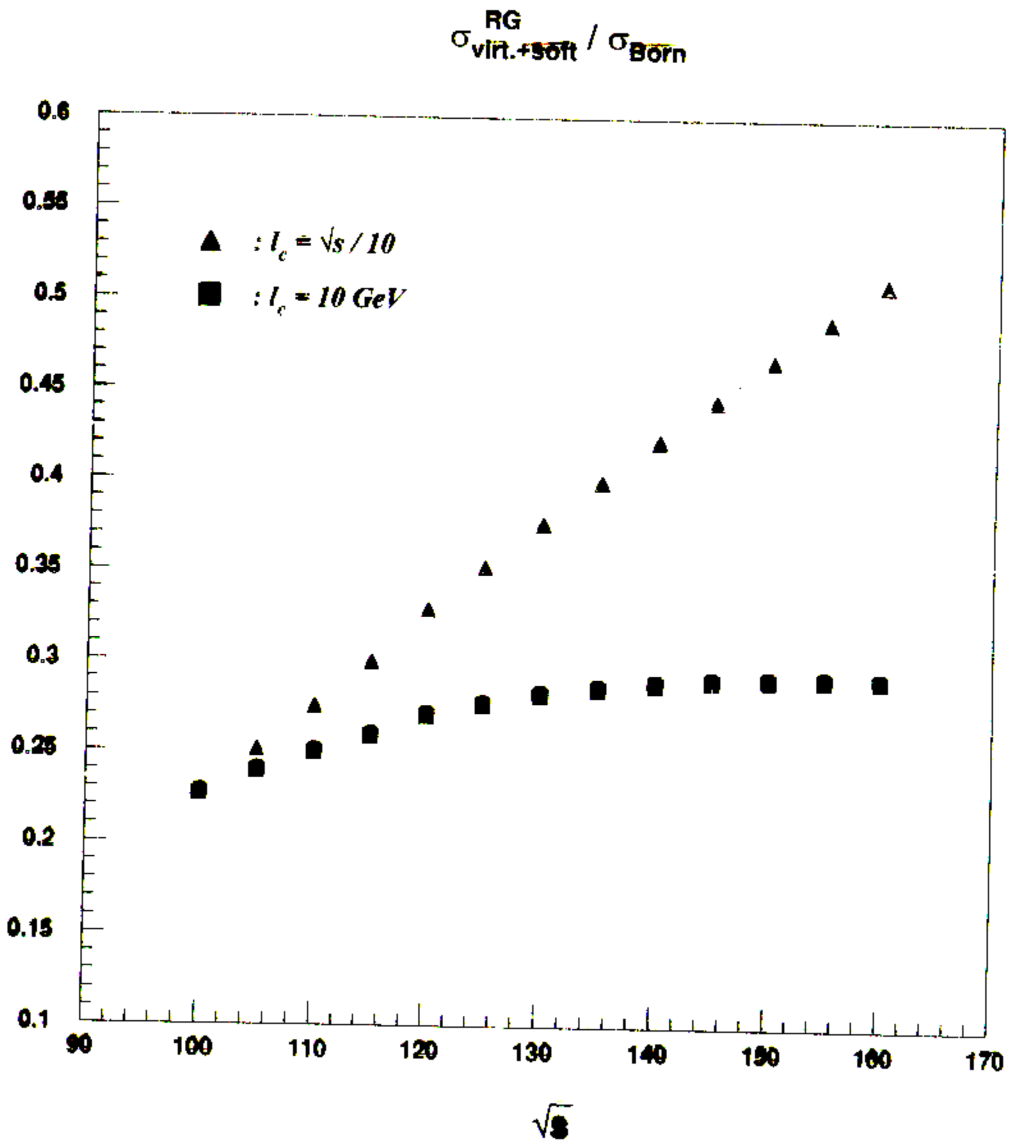


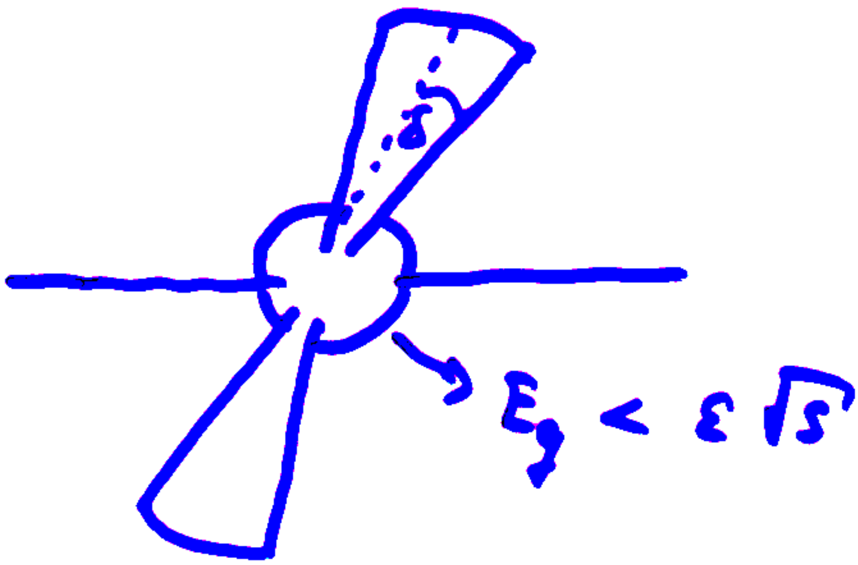
Figure 7: The effect of incorporating a running coupling constant at each loop integration (circles) according to Eq. 16. The one- and two-loop running coupling solutions are within 8% of each other. Also shown are upper and lower limits according to the indicated values of α_s in the DL approximation.

For full c.s., we need also the Sudakov F.F. & the 1 loop hard Bremsstrahlung (2 jet) contribution


σ^RG_virt.+soft / σ_Born
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
: l_c = √s / 10
: l_c = 10 GeV
90
100
110
120
130
140
150
160
170
√s

Include collinear hard Bremsstrahlung.

Define 2 jet cross section (Sterman, Weinberg):

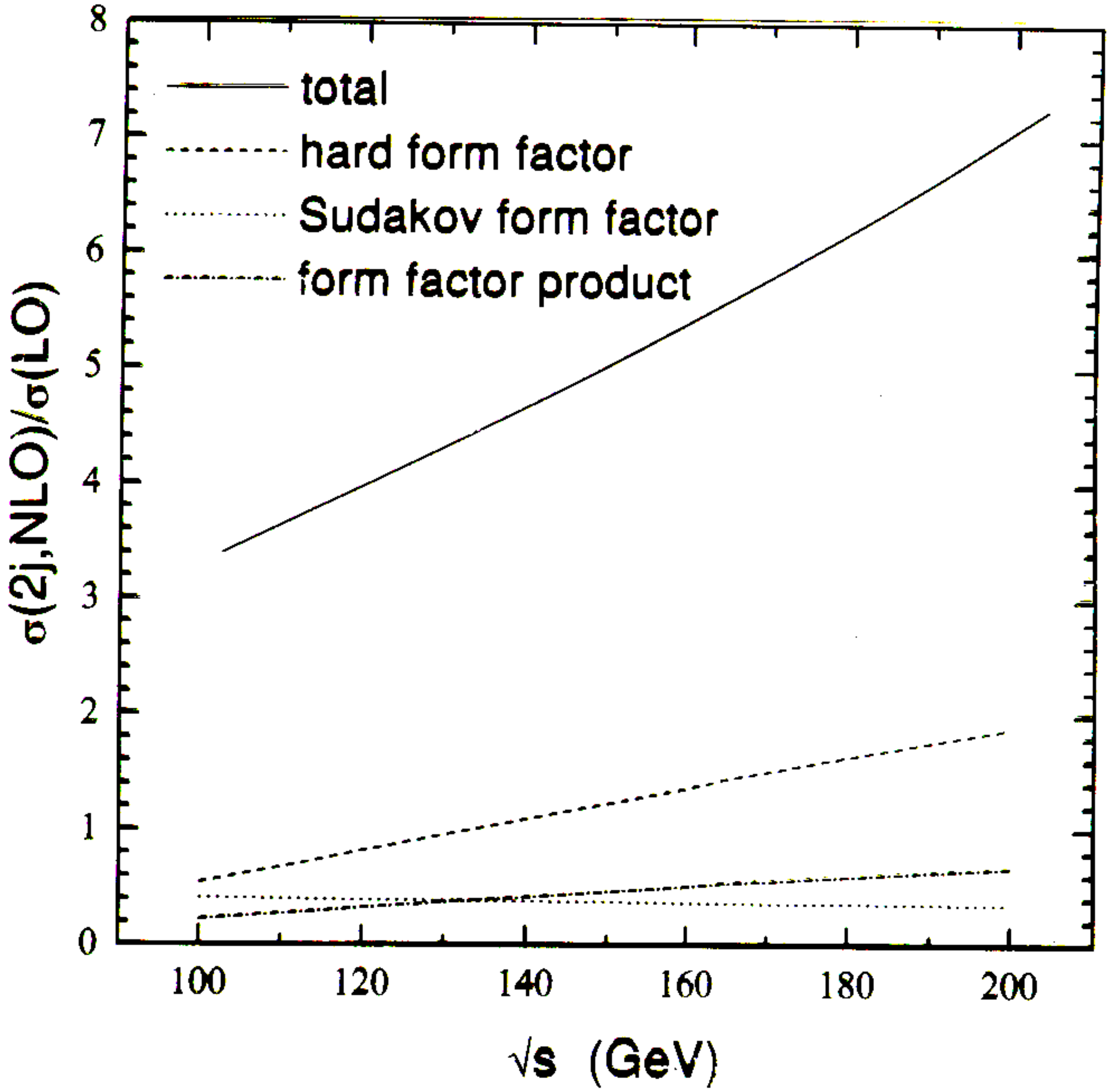


For all orders resummed result, try to reduce dependence on higher order l_c in the Sudakov F.F.

$\rightarrow$ choose $l_{c/\sqrt{s}} = \varepsilon = 0.1$

here: $\delta = 20^\circ$

$\frac{l_z}{\sqrt{s}} = \varepsilon = 0.1$



→ reduce collinear phase space by choosing narrower jet def. ($\delta < 20^\circ$)

Summary:

- We have derived the RG improved massive Sudakov F.F. (Real + Virt.)
- The non-Sudakov F.F. is also RG improved (eff. scale ~ $q\, m^2$)
- Effect of scale fixed results is significant (almost factor 2)

→ narrow jet definition can reduce BG significantly! (but also signal!)

Next: Full MC study including:
- "charm pollution"
- signal/BG as a function of jet def.
- detector efficiencies

⋮