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One-loop QCD interconnection effects
in
top-pair production

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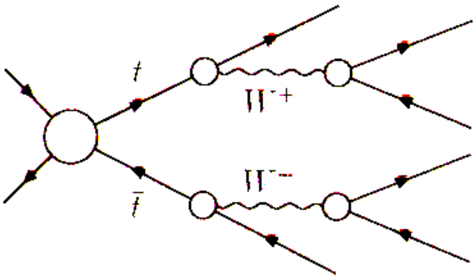
1. **Introduction**
2. **Factorizable vs. Non-factorizable**
3. **Calculations**
4. **Small β behaviour**
5. **Results and Conclusions**

Various contributions

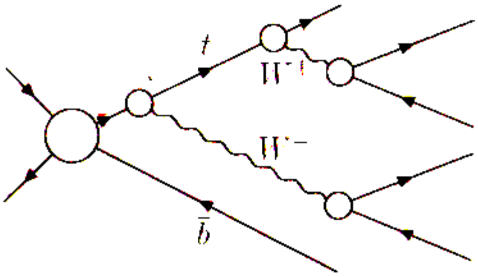
- **Pole scheme.**
  Expansion in $1/D_{1,2}$, with $D_{1,2} = M_{1,2}^2 - M_t^2 + i\Gamma_t M_t$.

- Structure of matrix element

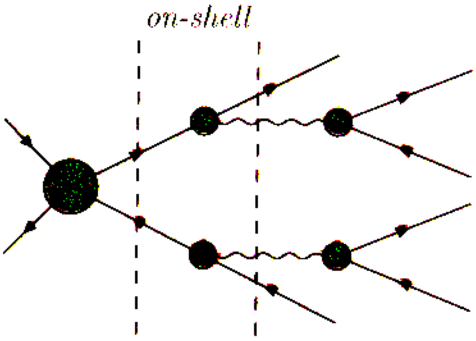
$$\mathcal{M} \sim \frac{1}{D_1 D_2} + A\frac{\Gamma_t}{M_t} + \alpha_S \frac{B(M_t; M_t)}{D_1 D_2} + \alpha_S \frac{C(D_1; D_2)}{D_1(D_1 + D_2)}$$



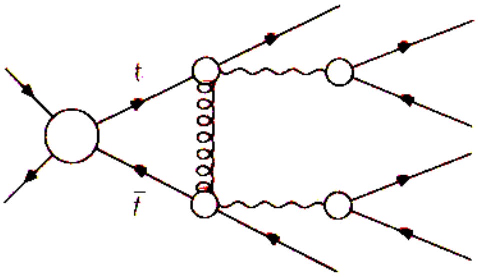
Born



background



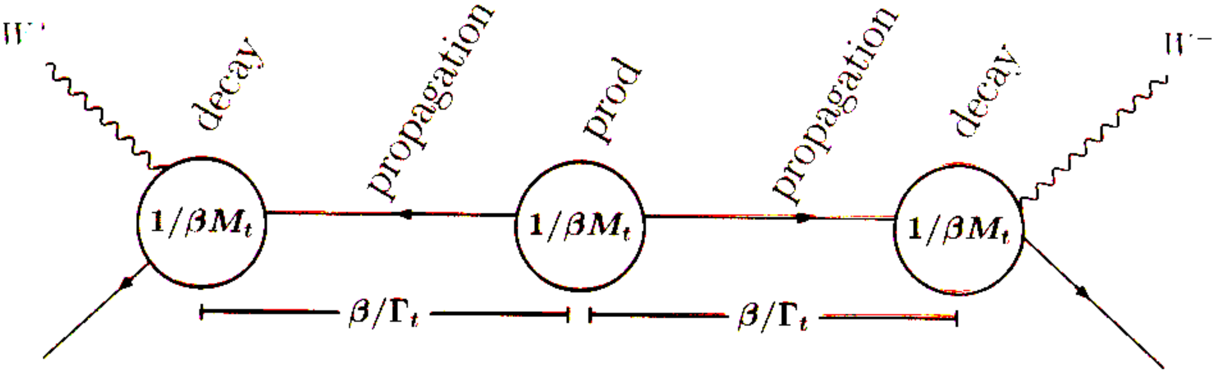
radiative factorizable



radiative non-factrizable

- ○ neglected terms $\mathcal{O}\left(\alpha_S^2;\ \alpha_S \frac{\Gamma_t}{M_t};\ \frac{\Gamma_t^2}{M_t^2};\ \alpha\right)$
- ○ Note that $\Gamma_t/M_t \approx \alpha$.

Factorizable vs Non-factoriable corrections



○ far from threshold($\Delta E \approx M_t$, $\beta \sim 1$)

factorizable: $\boldsymbol{\omega_g \approx M_t}$
non-factorizable: $\boldsymbol{\omega_g \approx \Gamma_t}$

○ close to threshold ($\Delta E \gg \Gamma_t$, $\beta \ll 1$)

factorizable: $\boldsymbol{\omega_g \gtrsim \beta^2 M_t \quad |k_g| \gtrsim \beta M_t}$
non-factorizable: $\boldsymbol{\omega_g \sim \Gamma_t \qquad |k_g| \sim \Gamma_t/\beta}$

- accuracy of the calculation

○ $M_t \approx \Delta E \gg \Gamma_t \quad \rightarrow \mathcal{O}(\Gamma_t/M_t)$
○ $M_t \gg \Delta E \gg \Gamma_t \quad \rightarrow \mathcal{O}(\Gamma_t/\Delta E)$
○ threshold: $\Delta E \approx \Gamma_t \quad \rightarrow \mathcal{O}(1)$ method breaks down

Non-factorizable corrections

- Virtual

$$d\sigma_{\rm nf}^{\rm virt} \sim 2{\rm Re}\ i \int \frac{d^4k}{(2\pi)^4k^2}\Big[\mathcal{J}_0\mathcal{J}_+ + \mathcal{J}_0\mathcal{J}_- + \mathcal{J}_+\mathcal{J}_-\Big]$$

with

$$\mathcal{J}_0^\mu = g\Big[\frac{p_1^\mu}{-kp_1} + \frac{p_2^\mu}{kp_2}\Big],$$

$$\mathcal{J}_+^\mu = -\,g\Big[\frac{p_1^\mu}{-kp_1} - \frac{k_1^\mu}{-kk_1}\Big]\frac{D_1}{D_1 - 2kp_1},$$

$$\mathcal{J}_-^\mu = -\,g\Big[\frac{p_2^\mu}{kp_2} - \frac{k_2^\mu}{kk_2}\Big]\frac{D_2}{D_2 + 2kp_2}$$

- Real

$$d\sigma^{\rm real} \sim -\int\frac{d\vec{k}}{(2\pi)^3 2\omega_g}\Big[\underbrace{2{\rm Re}\left(\mathcal{I}_0\mathcal{I}_+^* + \mathcal{I}_0\mathcal{I}_-^* + \mathcal{I}_+\mathcal{I}_-^*\right)}_{\text{non-factorizable}} + \underbrace{|\mathcal{I}_0^2| + |\mathcal{I}_+^2| + |\mathcal{I}_-^2|}_{\text{factorizable}}\Big]$$

with

$$\mathcal{I}_0^\mu = g\Big[\frac{p_1^\mu}{kp_1} - \frac{p_2^\mu}{kp_2}\Big]$$

$$\mathcal{I}_+^\mu = -\,g\Big[\frac{p_1^\mu}{kp_1} - \frac{k_1^\mu}{kk_1}\Big]\frac{D_1}{D_1 + 2kp_1}$$

$$\mathcal{I}_-^\mu = +\,g\Big[\frac{p_2^\mu}{kp_2} - \frac{k_2^\mu}{kk_2}\Big]\frac{D_2}{D_2 + 2kp_2}$$

- bremsstrahlung: not only $3p$, but $4p$ and $5p$ functions

Nf corrections: calculation

Some features

Fadin, Khoze, Martin, Stirling
Melnikov, Yakovlev

- Connection of real and virtual

$$d\sigma_{\text{real}} = -d\sigma^{g-\text{pole}}_{\text{virt}}\Big|_{D_1\to -D_1^*}$$

- Initial-Final state interference vanish
  no prod. angle dependence
- Zero upon integration over M_1 and M_2
- $\delta_{\text{nf}} \sim E^{-1}$ at high energies
  $(1-\beta)^2/\beta$-ansatz

Khoze and co-authors

Calculations

- Melnikov, Yakovlev
- Berends, Beenakker, Chapovsky
  2 methods
  one allows for more complicated processes
  decomposition of $Np-$ functions
- Denner, Dittmaier, Roth

now agree

agree for W^+W^-

Small β behaviour

- **Coulomb effect** *($1/\beta$-singularity)*
  - ○ soft gluons
  - ○ accuracy $\mathcal{O}(\beta)$

- **Non-factorizable correction**
  - ○ soft gluons
  - ○ accuracy $\mathcal{O}(\Gamma_t/\Delta E)$

- **Intersection region**

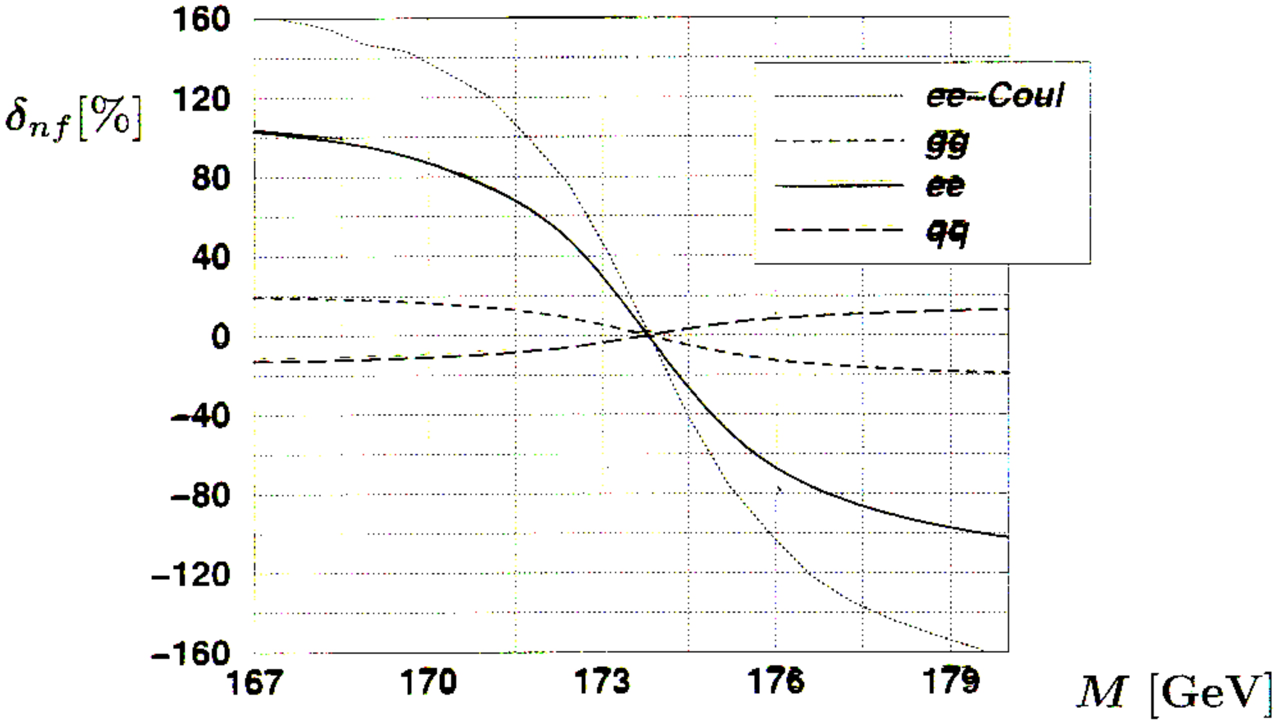
$$\Gamma_t \ll \Delta E, \quad \beta \approx \sqrt{\frac{\Delta E}{M_t}} \ll 1.$$

- **Off-shell Coulomb effect**

$$\delta_{\text{Coulomb}} = \frac{\alpha}{2\beta}\Big[\underbrace{\pi}_{\text{fact}} - 2\arctan\underbrace{\Big(\frac{D_1+D_2}{2\Gamma_t M_t}\Big)}_{\text{non-fact}}\Big]$$

Nf-correction to invariant mass distribution

at 355 GeV $\approx 2M_t + 5\Gamma_t$ $\quad (\beta \approx 0.2)$



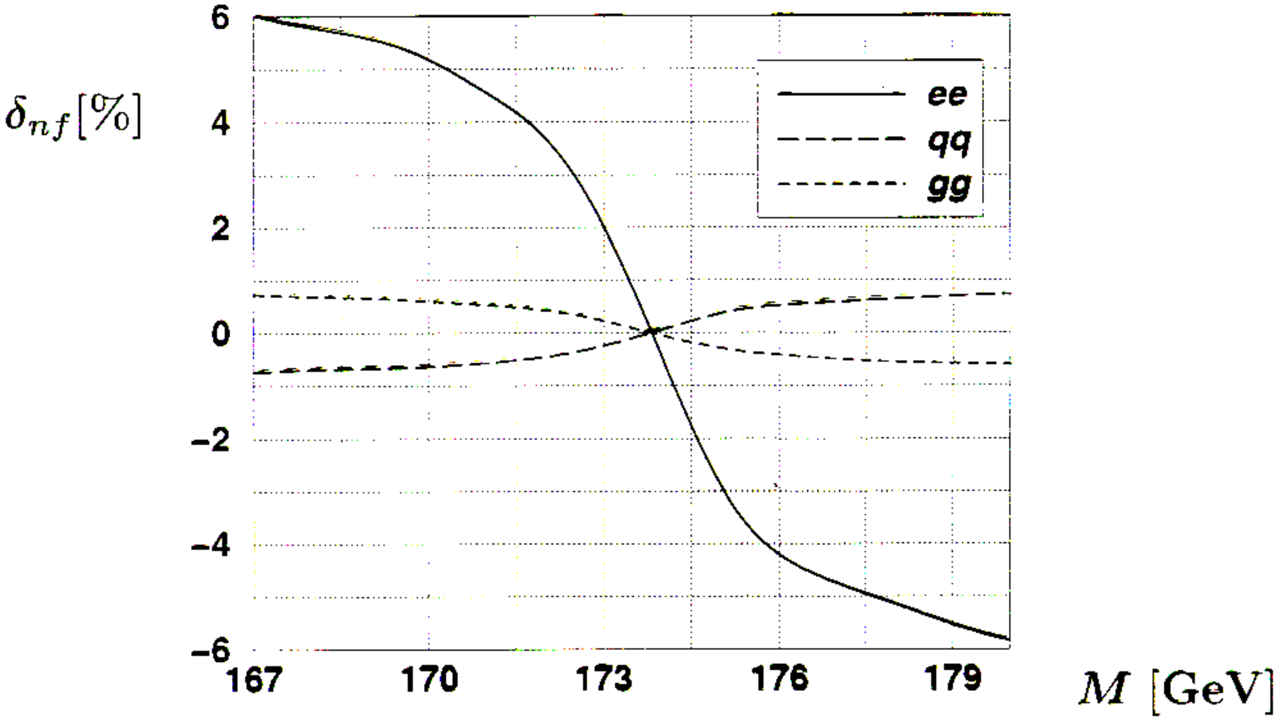
$$\frac{d\sigma}{dM_1} = \frac{d\sigma_{\rm Born}}{dM_1}(1 + \delta_{\rm nf}(M_1))$$

- Mass shift

$$\Delta M^{peak} = \frac{1}{8}\Gamma_W^2 \delta'_{\rm nf}(M_1)$$

$$\Delta M^{peak} \approx -85[+10]\text{MeV} \quad \text{for} \quad e^+e^- \quad \text{and} \quad \gamma\gamma \quad [q\bar{q}]$$

Nf-correction to invariant mass distribution at 500 GeV



- consistent with $\delta_{\mathrm{nf}} \sim \frac{(1-\beta)^2}{\beta}$

- **Mass shift**

$$\Delta M^{peak} \approx -5\mathrm{MeV} \quad \text{for} \quad e^+e^- \quad \text{and} \quad \gamma\gamma$$

Conclusions

- $\mathcal{O}(\alpha_S)_{\rm nf}$ to tt production
- $\mathcal{O}(\alpha)_{\rm nf}$ can be done
- $\mathcal{O}(\alpha_S^2)_{\rm nf}$ might be hard
- At threshold should be combined with resummations
- Other applications: W^+W^-, ZZ, Higgs, ...