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ECFA/DESY

Determining low-energy SUSY parameters from $e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-$

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1. The problem
2. $\sigma(e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ + $\tilde{\chi}_1^\pm$ polarization

   $\Rightarrow$ $M_2, \mu, \tan\beta$

3. $\sigma(e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-)$, A_{LR} $\quad i,j = 1, 2$

   $\Rightarrow$ $M_2, \mu, \tan\beta$

4. expected errors, role of $\mathcal{L} = 500\ fb^{-1}$
5. determining M_1

S.Y. Choi, A. Djouadi, H. Dreiner, P. Zerwas, H. Song, J.K. Collaboration start from the simplest case

charginos $M_2, \mu, \tan\beta$

neutralinos $+M_1$

sleptons $+ m_{\tilde{\ell}}, A_\ell$

squarks $m_{\tilde{q}}, m_{\tilde{q}}, A_q$

in each case

fundamental parameters $(M_i, \mu, \tan\beta, A_i ..)$

$\updownarrow$

physical parameters : masses couplings

$\updownarrow$

physical observables; masses, σ, A_{LR}

correlations etc.

$\Rightarrow$ determine $M_1, M_2, \tan\beta, \mu$ in

a model independent way!

2. DETERMINING THE STRUCTURE

Charginos

Tsukamoto et al.
Moortgat-Pick et al.
~~Feng~~ et al.
Choi et al.
Lafaye et al.
⋮

$W^\pm \to \tilde{W}^\pm$ gaugino

$H_1\ H_2 \to \tilde{H}_1, \tilde{H}_2$ higgsino

$\psi^\pm = \begin{pmatrix} -i\tilde{W}^\pm \\ \tilde{H}^\pm \end{pmatrix}$

mass term $\quad \psi^- M_c \psi^+ + h.c.$

$$M_c = \begin{pmatrix} M_2 & \sqrt{2}\, m_W \cos\beta \\ \sqrt{2}\, m_W \sin\beta & \mu \end{pmatrix}$$

$$\left\{ \begin{array}{l} M_2 - SU(2) \text{ gaugino mass} \\ \mu - \text{Higgs mixing parameter:} \quad \mu H_1 \cdot H_2 \\ \tan\beta = \dfrac{v_2}{v_1} = \dfrac{\langle H_2^0 \rangle}{\langle H_1^0 \rangle} \end{array} \right.$$

charginos $\tilde{\chi}_i^\pm$ mass eigenstates

$$m_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\left(M_2^2 + \mu^2 + 2m_W^2 \mp \sqrt{(M_2^2 + \mu^2 + 2m_W^2)^2 - 4(M_2\mu - m_W^2 \sin 2\beta)^2} \right)$$

$$\tilde{\chi}^-_{1L} = \tilde{W}^-_L \cos\phi_L + \tilde{H}_1^- \sin\phi_L$$

$$\tilde{\chi}^-_{1R} = \tilde{W}^-_R \cos\phi_R + \tilde{H}_2^- \sin\phi_R$$

$$M_2,\ \mu,\ \tan\beta \quad \Rightarrow \quad m_{\tilde{\chi}_1^\pm},\ \phi_L,\ \phi_R$$

$\Leftarrow$?

CP phases

M_2 real, $\quad \mu = |\mu|\, e^{i\theta_\mu}$

masses:

$$m^2_{\chi^\pm_{1,2}} = \frac{1}{2}\left(M_2^2 + |\mu|^2 + 2m_W^2 \mp \Delta\right)$$

$$\Delta = \left[\left(M_2^2 + |\mu|^2 + 2m_W^2\right)^2 - 4\left(M_2^2|\mu|^2 - 2m_W^2 M_2 |\mu| \sin 2\beta \cos\theta_\mu + m_W^4 \sin^2 2\beta\right)\right]^{1/2}$$

mixing angles:

$$\cos 2\phi_L = -\frac{M_2^2 - |\mu|^2 - 2m_W^2 \cos 2\beta}{\Delta}$$

$$\cos 2\phi_R = -\frac{M_2^2 - |\mu|^2 + 2m_W^2 \sin^2\beta}{\Delta}$$

couplings:

and

real

complex but

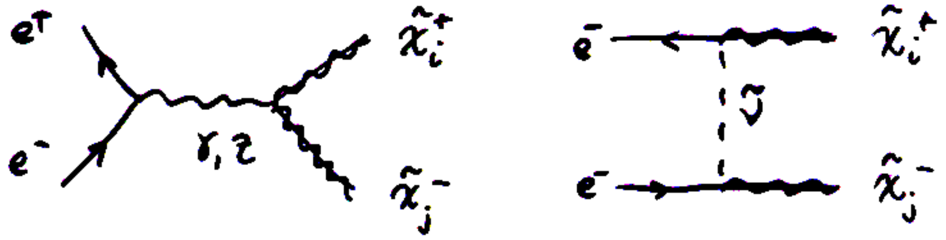
amplitude real

$M_2, \mu, \tan\beta \Longleftrightarrow m_{\chi_1^+},\ \cos 2\phi_L,\ \cos 2\phi_R \Longrightarrow$ amplitudes the same

depends on CP phases

phase independent

$e^+e^- \to \tilde{\chi}_i^+ \tilde{\chi}_j^-$



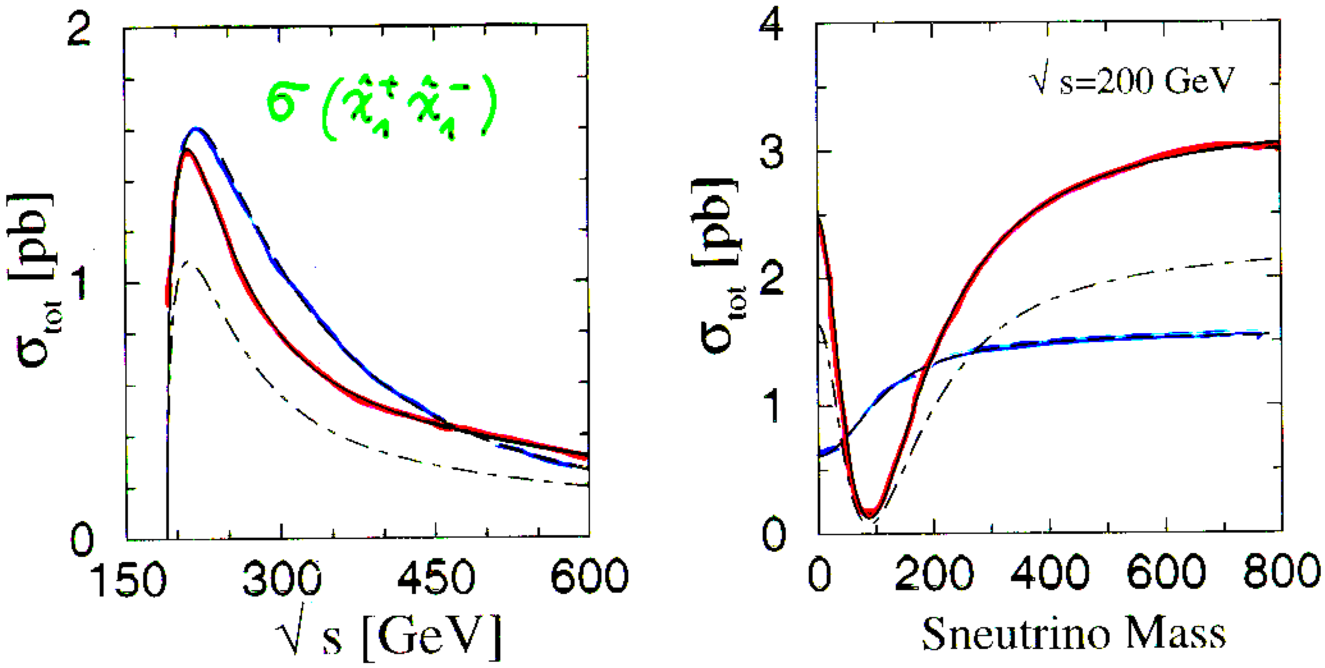
gaugino $M_2 = 81$, $\mu = -215$ GeV

higgsino $M_2 = 215$, $\mu = -81$ GeV

mixed $M_2 = 92$, $\mu = -93$ GeV

$m_{\tilde{\chi}_1^\pm} = 95$ GeV, $\tan\beta = 2$

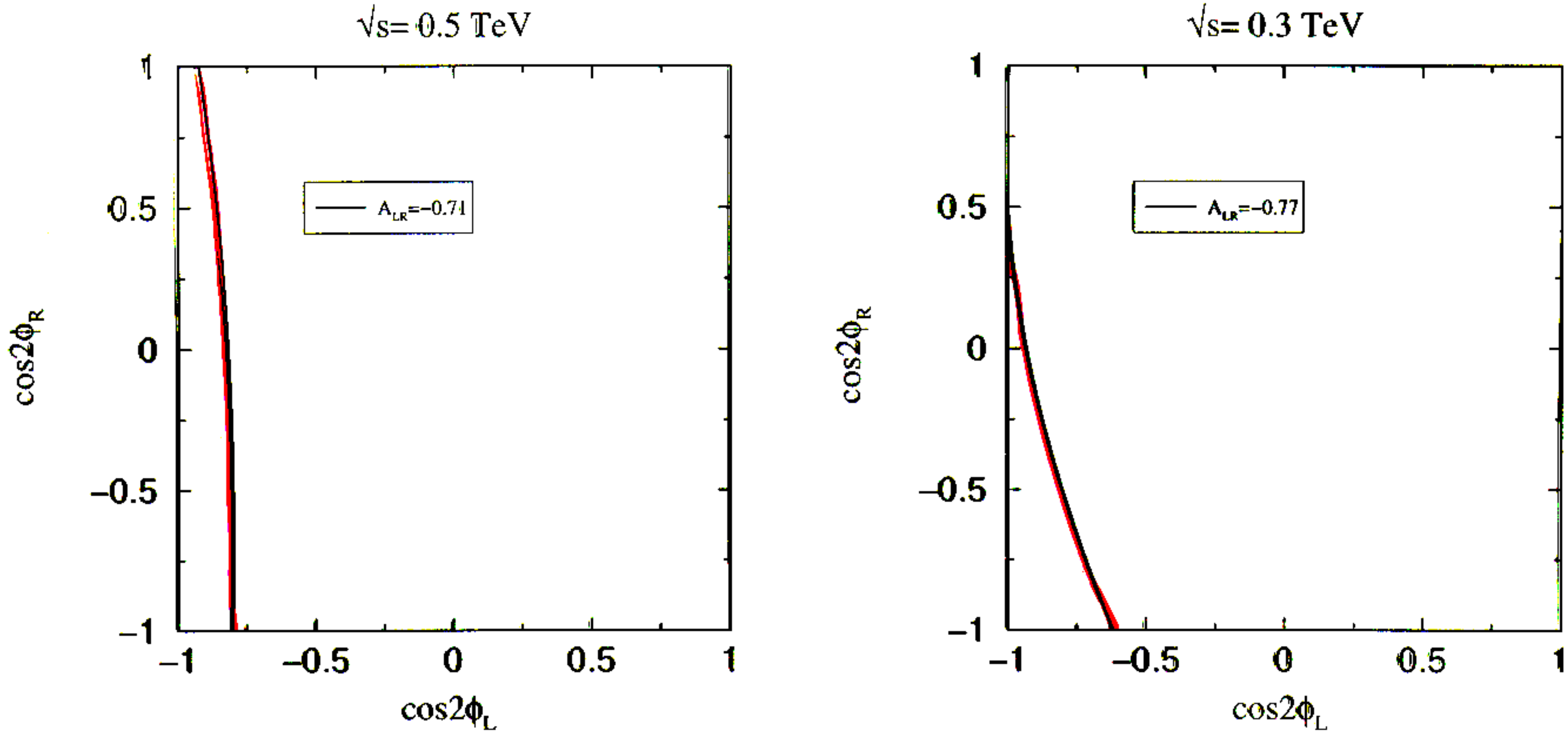
Choi et al.



(a) (b)

$m_{\tilde{\nu}} = 200$ GeV

add A_{LR} if available



unique solution $\Rightarrow$ $m_{\tilde{\chi}_1^+} = 95$ GeV

$\cos 2\phi_L = -0.8$

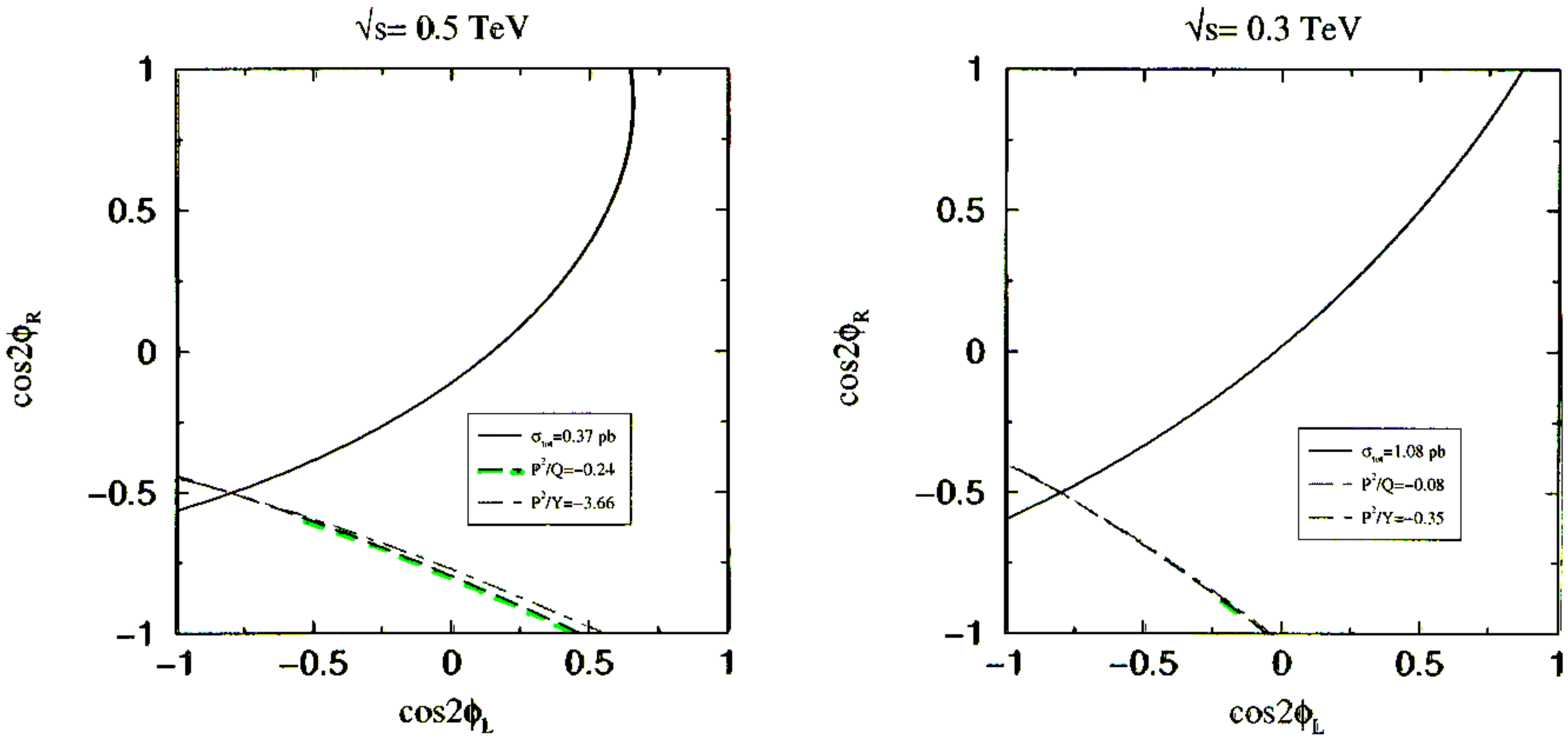
$\cos 2\phi_R = -0.5$

$\longrightarrow$ fundamental parameters

$\tan\beta$	1.06	3.33	
M_2	83	248	GeV
μ	−59	123	GeV

two-fold ambiguity

physical observables → physical parameters (vary energy as well)



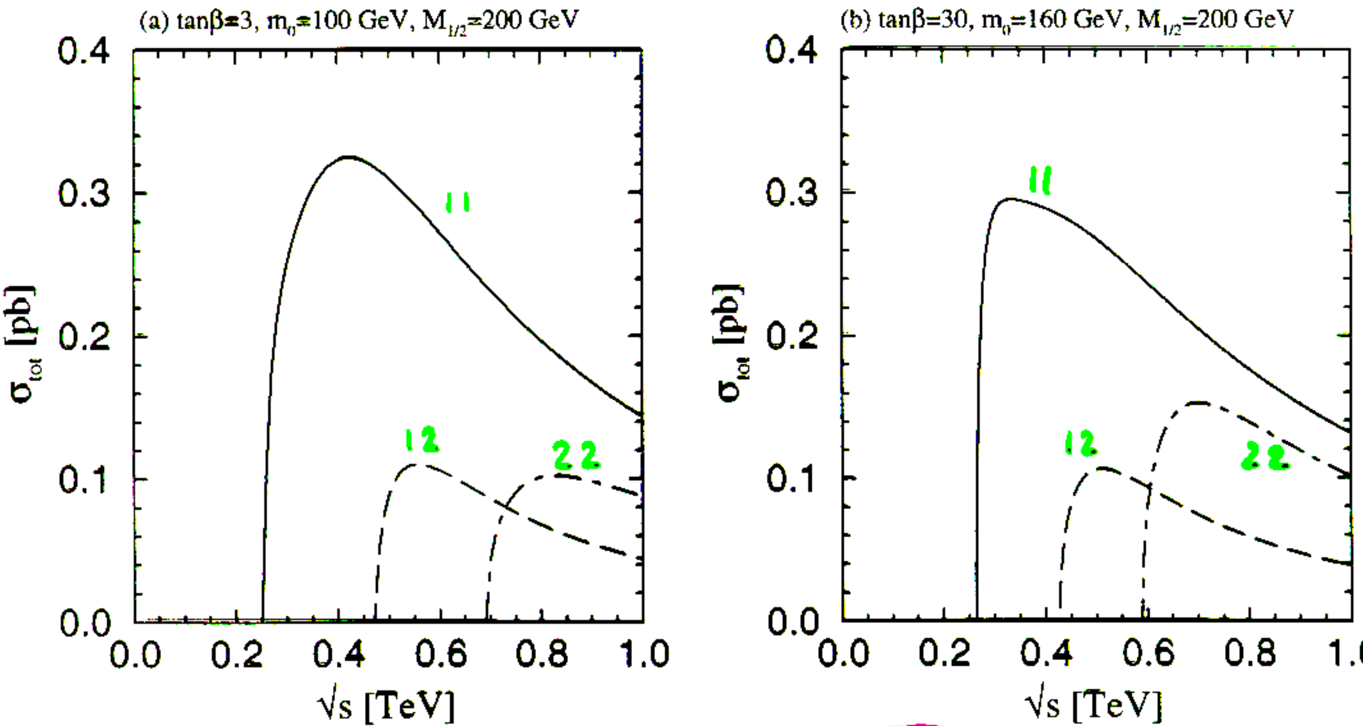
if $\sqrt{s}$ high enough $\rightarrow$ exploit $\chi_1^+\chi_2^-$, $\chi_2^+\chi_2^-$

measure:

* $m_{\tilde{\chi}_1}$, $m_{\tilde{\chi}_2}$ from threshold behaviour

* L-R asymmetry for 11, 12 and 22

Choi, Djouadi, Song, Zerwas



RR1

$M_2 = 152$ $\tan\beta = 3$

$\mu = 316$ $m_{\chi^\pm} = 128, 346$

$m_{\tilde{\nu}} = 166$ $m_{\chi_1^0} = 70$

RR2

$M_2 = 150$ $\tan\beta = 30$

$\mu = 263$ $m_{\chi^\pm} = 132, 295$

$m_{\tilde{\nu}} = 206$ $m_{\chi_1^0} = 72$

Choi, Djouadi, Song, Zerwas

$\sqrt{s} = 800$ GeV

RR1

$\sigma_t = 0.197$ pb

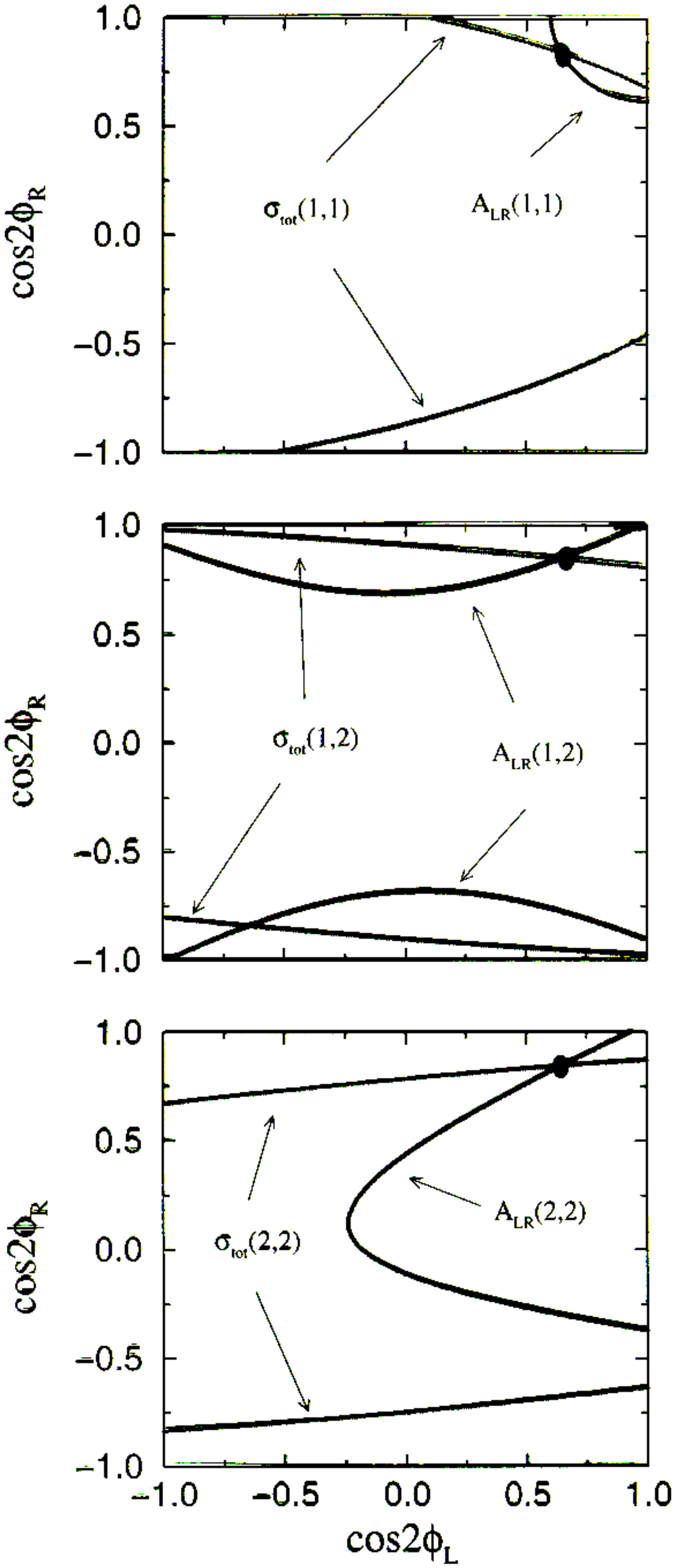
$A_{LR} = -0.995$

$\sigma_t = 0.068$ pb

$A_{LR} = -0.911$

$\sigma_t = 0.101$ pb

$A_{LR} = -0.668$



unique solution for $\{m_{\chi_1}, m_{\chi_2}, \cos 2\phi_L, \cos 2\phi_R\}$

and unique solution for fundamental parameters

$\{\tan\beta, M_2, \mu\}$

Fundamental susy parameters

$m_{\chi_1^+}$, $m_{\chi_2^+}$, $\cos 2\phi_L$, $\cos 2\phi_R$

(1) $$\tan\beta = \left[\frac{4m_W^2 + (m_{\chi_2^+}^2 - m_{\chi_1^+}^2)(\cos 2\phi_R - \cos 2\phi_L)}{4m_W^2 - (m_{\chi_2^+}^2 - m_{\chi_1^+}^2)(\cos 2\phi_R - \cos 2\phi_L)} \right]^{1/2}$$

and $(M_2 > 0)$

(2) $$\begin{cases} M_2 = \frac{1}{2}\left[2(m_{\chi_2^+}^2 + m_{\chi_1^+}^2 - 2m_W^2) - (m_{\chi_2^+}^2 - m_{\chi_1^+}^2)(\cos 2\phi_R + \cos 2\phi_L) \right]^{1/2} \\ |\mu| = \frac{1}{2}\left[\quad '' \quad + \quad \right]^{1/2} \end{cases}$$

and sign of μ

(3) $$\mathrm{sign}(\mu) = \frac{(m_{\chi_2^+}^2 - m_{\chi_1^+}^2)^2 - (M_2^2 - \mu^2)^2 - 4m_W^2(M_2^2 + \mu^2) - 4m_W^4\cos^2 2\beta}{8m_W^2 M_2|\mu| \sin 2\beta}$$

$\Rightarrow$ unique solution for $\tan\beta$, M_2, μ.

experimental errors ??

two reference points

	RR1	RR2
$\tan\beta$	3	30
m_0	100	160
$M_{1/2}$	200	200
M_2	152	150
μ	316	263
$\chi_1^\pm$	128	132
$\chi_2^\pm$	346	295
χ_1^0	70	72
$\tilde{\nu}$	166	206

take 1σ error on σ, A_{LR}

$\mathcal{L} = 500\ fb^{-1}$

$$\Rightarrow \quad \Delta(\cos 2\phi_L) = 0.02$$

$$\Delta(\cos 2\phi_R) = 0.005$$

assume $\Delta m_{\chi^\pm} \simeq 100$ MeV

$\Rightarrow$

	RR1	RR2
M_2	152 ± 1.8	150 ± 1.2
μ	316 ± 0.9	263 ± 0.7
$\tan\beta$	3 ± 0.7	30 ± 380 !

neutralinos do not help

σ_{11}, σ_{12}, σ_{22}

$m_{\chi_i^0}$, unless $\Delta m_{\chi_1^0} \simeq 100$ MeV ??

Why difficult to get $\tan\beta$?

– charginos + neutralinos

symmetric under $\beta \leftrightarrow \frac{\pi}{2} - \beta$

$\Rightarrow$ $\sin 2\beta$, $\cos 2\beta$

for large $\tan\beta$ $\Rightarrow$ $\sin 2\beta \simeq 0$, $\cos 2\beta \sim -1$

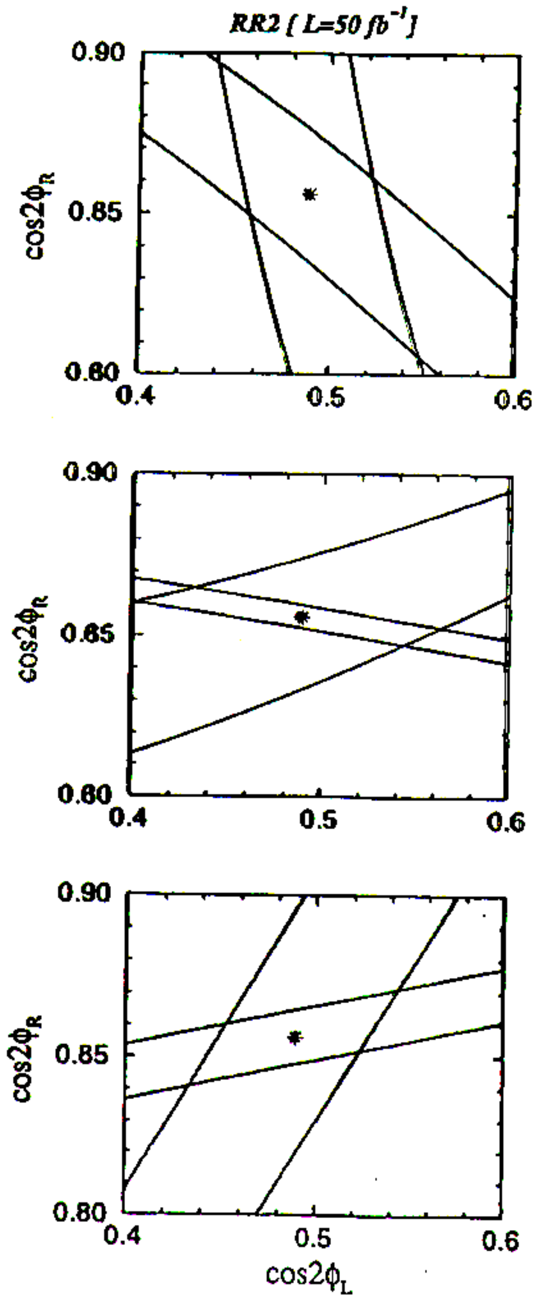
Different strategy: determine M_2, μ from (2)

$\cos 2\beta$ from (1)

$\text{sign}(\mu)$ + $\sin 2\beta$ from (3)

$\Rightarrow$ calculate $\tan\beta$

for RR2 : $\tan\beta = 30^{+30}_{-10}$

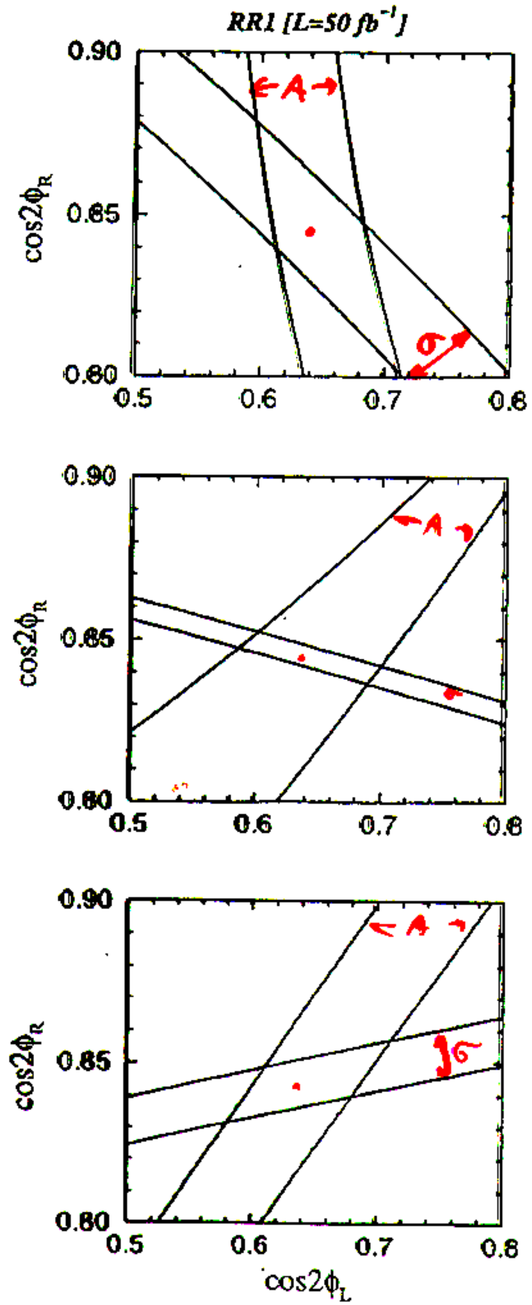


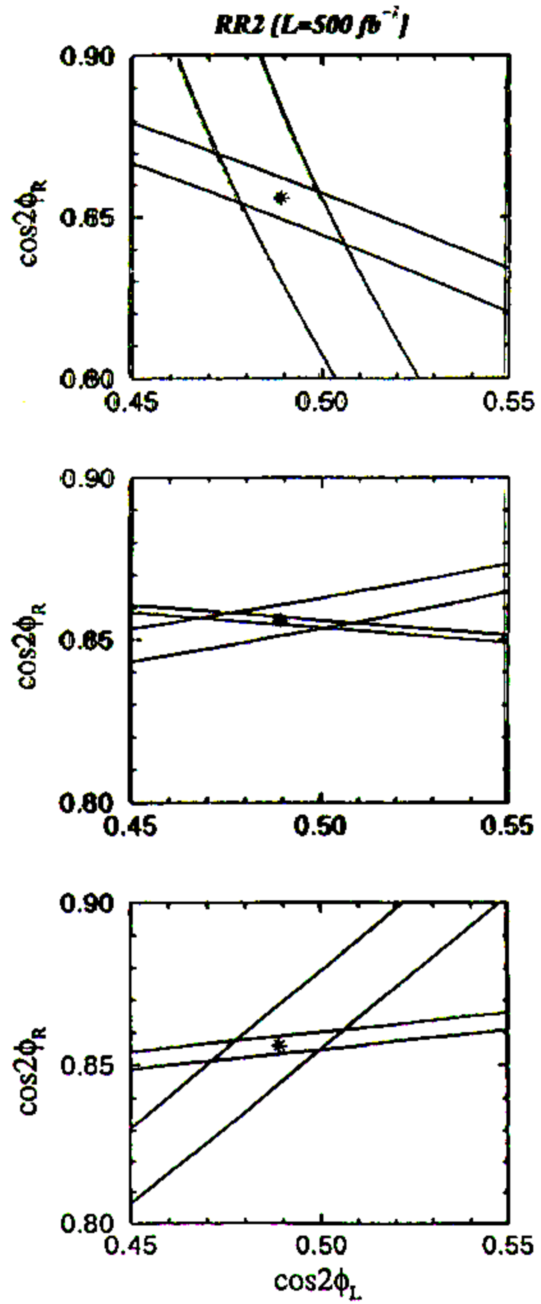
$L = 50\ fb^{-1}$

$\Delta \cos 2\phi_R \sim 0.02$

$\Delta \cos 2\phi_L \sim 0.1$

+49 40 89982777



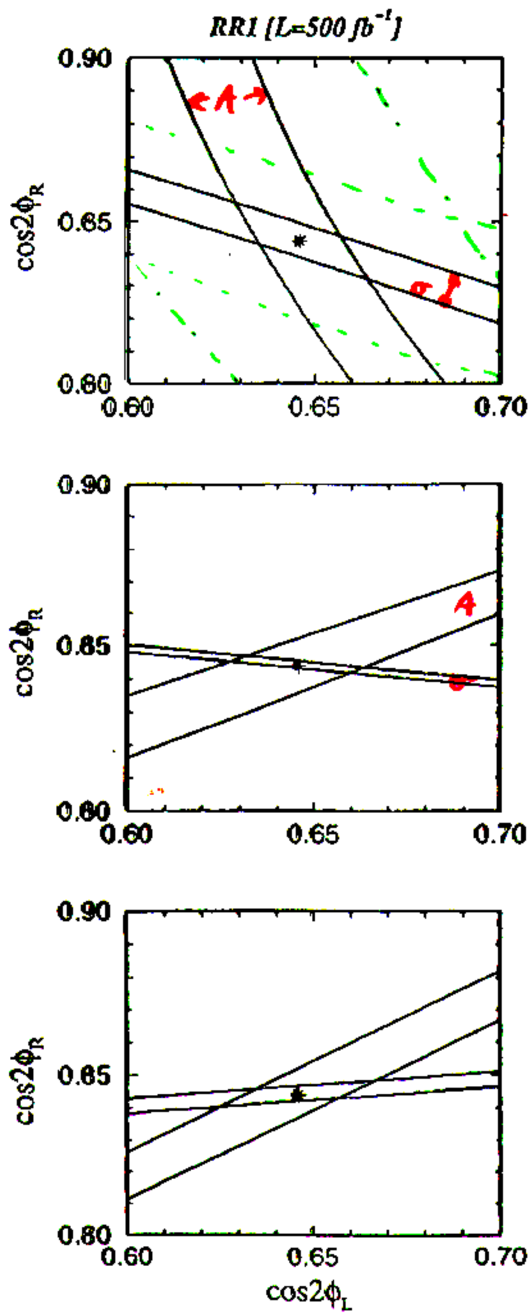


$L = 500\ fb^{-1}$

$\Delta \cos 2\phi_R \sim 0.005$

$\Delta \cos 2\phi_L \sim 0.02$

+49 40 89982777



···· σ
–·– A @ 50 fb^{-1}

← note the different range.

Determining M_1 from M_2, $\tan\beta$, μ, $m_{\tilde{\chi}_1^0}$

Mass matrix: $M_N = f(M_1, M_2, \beta, \mu)$

define

$$A = \mathrm{Tr}\, M_N$$

$$B = \frac{1}{2}\left[(\mathrm{Tr}\, M_n)^2 - \mathrm{Tr}\, M_n^2\right]$$

$$C = \frac{1}{6}\left((\mathrm{tr}\, M_N)^3 - 3\, \mathrm{Tr}\, M_N^2\, \mathrm{Tr}\, M_n + 2\, \mathrm{Tr}\, M_N^3\right)$$

$$D = \det M_N$$

$\rightarrow$ $m_{\tilde{\chi}_1^0} = x$ $\qquad \Delta m = 20\,\mathrm{MeV}$

$$x^4 - A x^3 + B x^2 - C x + D = 0$$

solve for M_1

Errors	$\mathcal{L} = 50\ fb^{-1}$	$\mathcal{L} = 500\ fb^{-1}$
RR1 ($\tan\beta = 3$)		
$m_{\tilde{\chi}^\pm} = 128, 345$ GeV	0.1 GeV	0.1 GeV
$\cos 2\phi_L = 0.65$	0.12	0.02
$\cos 2\phi_R = 0.84$	0.02	0.005
$M_2 = 152$ [GeV]	± 10.3	1.75
$\mu = 316$ [GeV]	4.96	0.85
$\cos 2\beta = -0.8$	0.49	0.08
$\sin 2\beta = 0.6$	0.34	0.058
$\tan\beta = 3$	(1.4 – 7.6)	(2.7 – 3.4)
$M_1 = 76$ [GeV]	3.5	0.64
(−67)	2.4	0.45

Errors

	$\mathcal{L} = 50\, fb^{-1}$	$\mathcal{L} = 500\, fb^{-1}$
RR 2 ($\tan\beta = 30$)		
$m_{\tilde{\chi}^{\pm}} = 132,\ 295$ [GeV]	0.1	0.1
$\cos 2\phi_L = 0.49$	0.12	0.02
$\cos 2\phi_R = 0.86$	0.02	0.005
$M_2 = 150$ [GeV]	7.03	1.20
$\mu = 263$ [GeV]	4.0	0.68
$\cos 2\beta = -0.9978$	0.33	0.056
$\sin 2\beta = 0.0666$	0.19	0.033
$\tan\beta = 30$	$(7.6 - \infty)$	$(20.2 - 59.6)$
$M_1 = 75.4$	1.96	0.40
(-74)	1.46	0.32

large luminosity crucial for $\tan\beta$!