

SYMMETRIES & CONSERVATIONS LAWS

Homework

If you have problems, do not hesitate to contact me:

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Homework Set 1 – Lectures 1 & 2

Q 1.1) By considering the first few terms of the expansions, prove that
$$\exp(A)\exp(B) \neq \exp(A+B)$$

if A and B do not commute.

However, show that the equality holds if A and B do commute (prove in general, to all orders).

Q 1.2) By expanding the exponential, find an expression for $\exp(i\alpha A)$ where $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

Q 1.3) If $[A,B]=B$, find an expression for $\exp(i\alpha A)B\exp(-i\alpha A)$ (consider to all orders).

Q 1.4) Consider which of the following are groups:

- Integers under Addition
- Integers under Subtraction
- Integers under Multiplication
- Reals under Multiplication

Any violations of the requirements for a group mean that the set and operation do not form a group.

Q 1.5) Demonstrate that there is only one group combination table for 3 distinct objects, i.e. all groups for 3 objects have the same form (are isomorphic) to Z_3 .

Do this by considering all the combinations for 3 distinct objects {e,a,b}.

Q 1.6) Show that the set of Lorentz Transformations:

$$g(\beta) \begin{cases} x' = \gamma(x - \beta t) \\ t' = \gamma(t - \beta x) \\ \gamma = 1/\sqrt{1 - \beta^2} \end{cases} \quad |\beta| < 1$$

form an Abelian Lie group under the operation “follows”.

Hint: start by combining two boosts: $g(\beta_2) g(\beta_1)$ and showing that these correspond to a third boost. Do this in Euclidean space. Easiest to employ matrix notation.

Homework Set 2 – Lectures 2 & 3

Q 2.1) Show that $U(n)$ and $SU(n)$ are groups.

Q 2.2) Consider rotations in 3D about the x-, y- and z-axes – $SO(3)$.

Identify generators appropriate to

a) Scalar wavefunctions $\psi(x)$ – we have done this in the Lectures; you can just write down the QM operators (do not write lots of blah, just write down operators)

b) Real vectors in 3D space – consider infinitesimal rotation matrices; the generators will be 3×3 matrices (give the rotation matrices, consider small angles and identify generators)

In both cases, find the structure constants. (Don't work out every single possibility, but appeal to symmetry.)

Q 2.3) For the generators $\{L_x, L_y, L_z\}$ in question 2.2, part (b), find the simultaneous eigenvectors of L^2 and L_z (i.e. the one set of vectors which are eigenvectors of both operators).

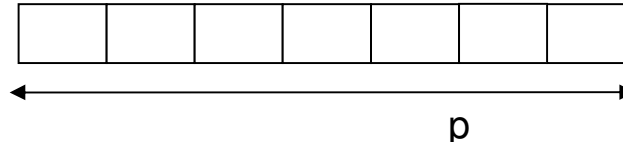
Q 2.4) Find the adjoint matrices for the generators in question 2.2, part (b).

In this case, it is obvious that they satisfy the Lie algebra.

Q 2.5) Verify the form of $R_y(\theta)$ for spin-1 given in Lecture 3.

Homework Set 3 – Lectures 4 & 5

Q 3.1) Using the Young Tableaux rules, write down the multiplicity for p particles of $SU(n)$ in a totally symmetric state, namely a row of p boxes:



Now consider examples of how the corresponding states could be labelled by supplying quantum numbers $\{1, 2, \dots, n\}$.

1	1	1	1	1	1	1
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1	1	1	1	1	1	2
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1	1	1	1	1	2	2
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By considering all the possible configurations, verify the multiplicity. Do this in general, not for a specific example.

Hint: This is *so* trivial, that it requires *no* algebra, but you have to spot the trick!

The trick is to consider the number of ways of listing p boxes with $(n-1)$ transitions of state label.

Q 3.2) Using the Young Tableaux rules, verify that the multiplicity of a general multiplet in $SU(2)$ is $(a+1)$ and in $SU(3)$ is $\frac{1}{2}(a+1)(b+1)(a+b+2)$.

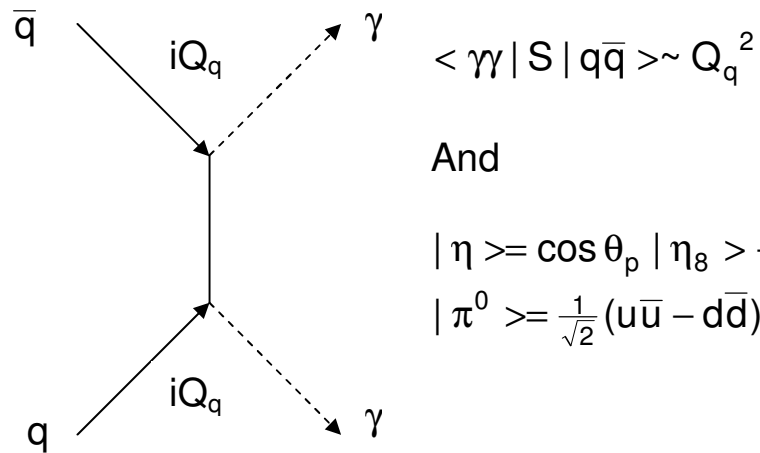
Q 3.3) Considering only flavour, find the ratio of matrix elements for $\pi^0 \rightarrow \gamma\gamma$ and $\eta \rightarrow \gamma\gamma$.
 Do this for a general case of mixing angle θ_p , and then choose θ_p such that the η has no strange-quark content.

Is the -ve sign in the π^0 wavefunction meaningful ?

How to proceed:

Label the scattering operator S and the meson state $|M\rangle$.

What you need is $\langle \gamma\gamma | S | M \rangle = \sum_q \langle \gamma\gamma | S | q\bar{q} \rangle \langle q\bar{q} | M \rangle$



And

$$|\eta\rangle = \cos\theta_p |\eta_8\rangle - \sin\theta_p |\eta_1\rangle$$

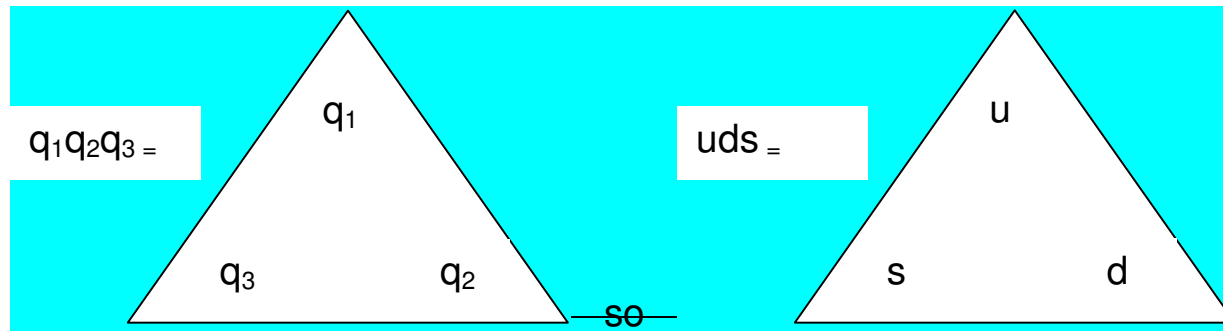
$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \text{ etc}$$

Q 3.4) IGNORE Express the flavour-spin wavefunction for the Λ baryon in an explicitly symmetric way, as we did for the proton.

$$\phi_{M,S} = \frac{1}{2} \{ (sd + ds)u - (su + us)d \} \quad \text{and} \quad \phi_{M,A} = \frac{1}{\sqrt{12}} \{ (sd - ds)u - (su - us)d - 2(du - ud)s \}$$

It is tricky to really make the wavefunction look symmetric due to the limitations of notation and writing on a page.

To help, I devised a simple notation:



And

