

1. Starting with the Lorentz Force Law in SI units, derive the formula for cyclotron motion:

$$p \text{ (GeV}/c) = 0.3qBR \quad (1)$$

2. The LHC is a 27 km circumference circular synchrotron with a 7.5 TeV proton beam travelling in one direction (and another 7.5 TeV proton beam travelling in the opposite direction)¹. It is the same size as LEP which had a 104 GeV electron beam (and a 104 GeV positron beam).

- What magnetic field strength was required at LEP to keep the electrons rotating in the ring?
- What magnetic field strength is required at the LHC to keep the protons rotating in the ring?
- At what speed is the proton travelling in the LHC (as a fraction of the speed of light)?
- Calculate the energy loss due to synchrotron radiation per turn per proton in the LHC beam in MeV (assume the proton is travelling at the speed of light).
- If the LHC has 1380 bunches and each bunch contains 2.0×10^{11} protons, how much energy is lost per second from synchrotron radiation?

3. The International Linear Collider (ILC) is a future linear collider designed to collide 500 GeV electrons on 500 GeV positrons.

- If the typical field strength of an accelerating cavity is 10 MV/m, how long must the ILC be?

4. Consider a highly relativistic proton beam with momentum $p = 500 \text{ GeV}/c$ incident on a hydrogen target.

- Calculate \sqrt{s} for the pp interaction (assume $m_p \approx 1 \text{ GeV}/c^2$).
- What beam energy would be required for a pp collider to achieve the same \sqrt{s} ? What does this say about the relative benefits of a fixed target and collider experiment?

5. The LHC has a 27 km circumference operating as a pp collider at $\sqrt{s} = 15 \text{ TeV}$ with 1380 bunches and 2.0×10^{11} protons per bunch. The average luminosity \mathcal{L} is currently $2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$.

- The strength of the beam is often expressed as a current, as if the beam-pipe was simply a wire with moving charges. What is the total beam current in the LHC in Amps? (remember each bunch can contribute multiple times per second).
- What is the total energy stored in the LHC beams, in Joules?
- What is the effective beam cross-section, in cm^2 ?
- Assuming that the LHC runs at the average luminosity for 23 hours each day, what will be the integrated luminosity after a month of running (in $\text{fb}^{-1} = 10^{-39} \text{ cm}^{-2}$).
- If the cross-section for producing a Higgs with mass $125 \text{ GeV}/c^2$ is 2 pb and the detection efficiency is 2%, how many Higgs bosons will be seen at the LHC in one month?

¹The LHC beam energy will be increased from 4.0 to 7.5 TeV in 2015