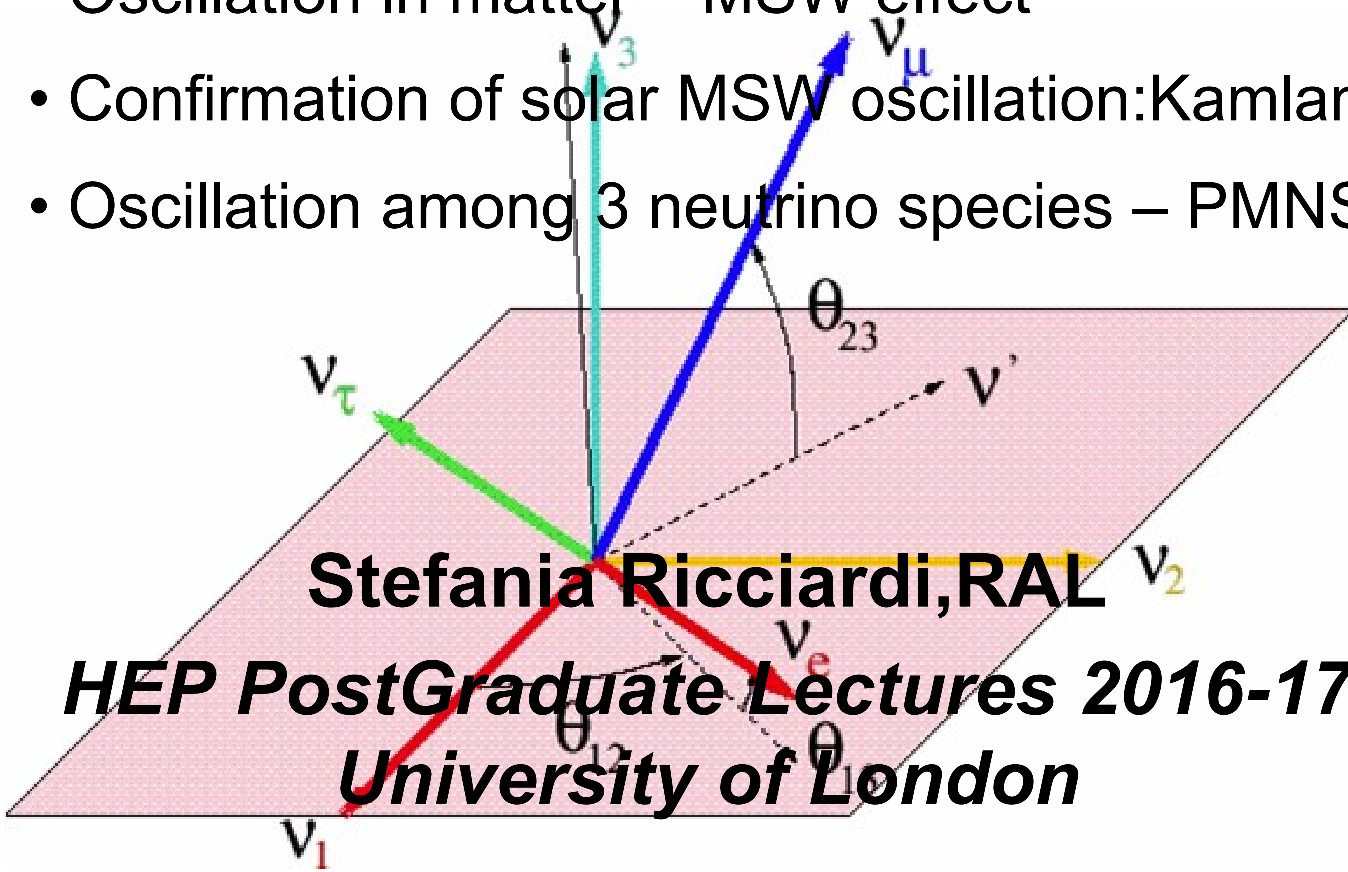
Unit 2: Phenomenology of Neutrino Mixing

- Oscillation in matter – MSW effect
- Confirmation of solar MSW oscillation:Kamland
- Oscillation among 3 neutrino species – PMNS matrix



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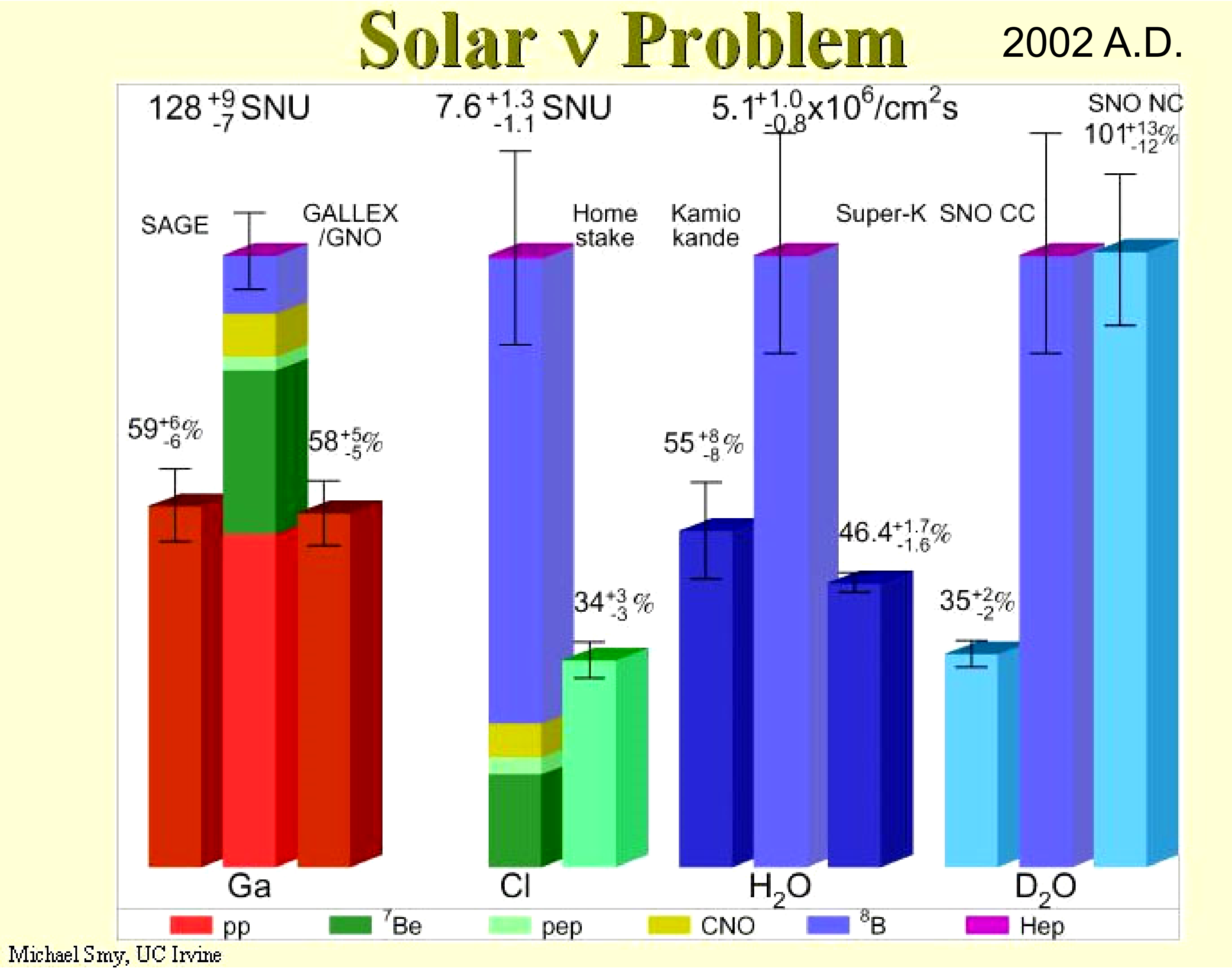


Again we need a energy dependent mechanism to explain all data

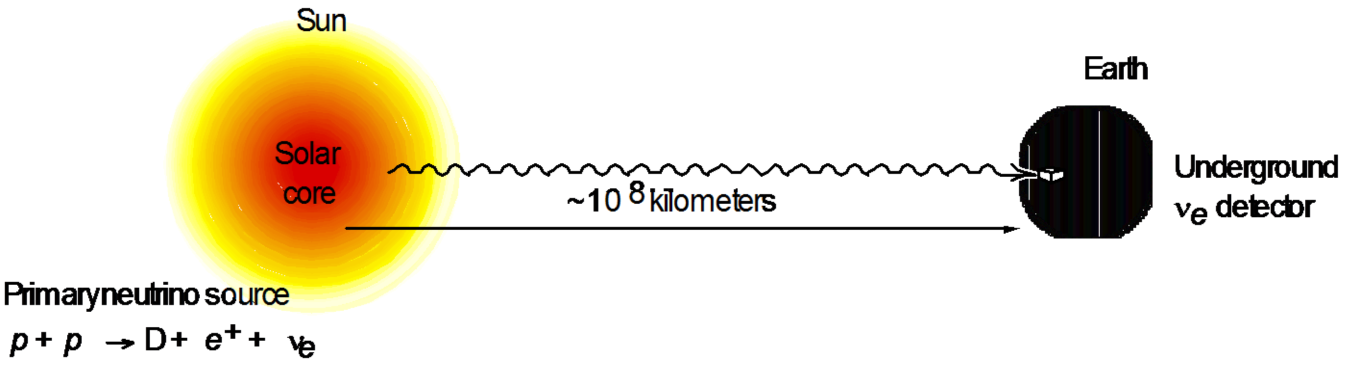
Is ν oscillation a good solution to all observed deficit?

Vacuum Oscillation

ν Flux reduction: Ga~40%, Cl~65%, water~50%

One possible explanation is that neutrinos oscillate in the propagation between Sun and Earth (vacuum oscillation)

The oscillation length needs to be “just right” so that ^8B and ^7Be neutrinos are depleted more than pp neutrinos



Survival Probability = $P(\nu_e \rightarrow \nu_e) = 1 - P(\nu_e \rightarrow \nu_x)$

$= 1 - \sin^2 2\theta \sin^2 (1.27\ \Delta m^2 (eV^2) L(m)/E(MeV))$

$\Delta m^2 (eV^2) L(m)/E(MeV) \sim 1 \Rightarrow \Delta m^2 \sim 10^{-11}\ eV^2$

Oscillations on long-baselines are sensitive to tiny mass differences!

Showing the potential of this quantum interference phenomenon.

The Hamiltonian for vacuum

Vacuum oscillation can be rewritten in the form of a Shroedinger equation

$|\nu_e(t)\rangle = \cos\theta\, e^{-iE_1 t}|\nu_1\rangle + \sin\theta\, e^{-iE_2 t}|\nu_2\rangle$
$|\nu_x(t)\rangle = -\sin\theta\, e^{-iE_1 t}|\nu_1\rangle + \cos\theta\, e^{-iE_2 t}|\nu_2\rangle$
Re-expressing $|\nu_1\rangle$ and $|\nu_2\rangle$ in terms of $|\nu_e\rangle$ and $|\nu_x\rangle$
$|\nu(t)\rangle = c_e(t)\,|\nu_e\rangle + c_x(t)|\nu_x\rangle$

Differentiating the coefficients, we obtain in matrix form

$$i\,d/dt \begin{pmatrix} c_e(t) \\ c_x(t) \end{pmatrix} = \pm\Delta \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} c_e(t) \\ c_x(t) \end{pmatrix}$$

$$i\,d/dt \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \mathcal{H} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}$$

the matrix plays the role of a Hamiltonian.

Where $\Delta=|\Delta m^2|/4E$, + sign applies if m2>m1, – sign if m2<m1

Note: adding multiple of identity matrix to H adds an overall phase which does not affect oscillating amplitudes

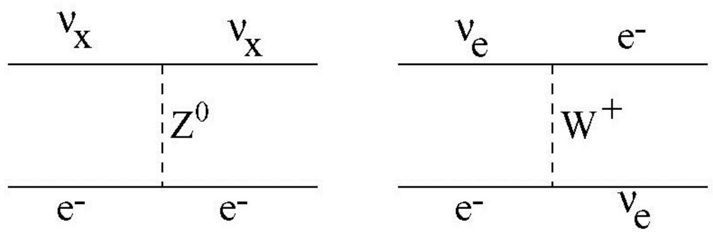
Matter Effects: MSW mechanism

MSW = Mikheyev – Smirnov - Wolfenstein

When neutrinos travel through matter (e.g. in the Sun, Earth, ..) their propagation is modified by scattering from particles they encounter along the way.
The interplay between flavor-nonchanging neutrino-matter interactions and neutrino-mixing can result in oscillation probability rather different than vacuum.

$$H = H_{Vacuum} + H_{Matter}$$

Charged-current ν_e elastic scattering singles out electron neutrinos



$$H = \pm\,\Delta \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \sqrt{2}\, G_F\, N_e \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

N_e = electron density, G_F = Fermi constant, $V_e = \sqrt{2}\, G_F\, N_e$ extra-potential "suffered" by ν_e (note for anti-νe the sign of V is reversed, so matter effects on oscillation distinguish ν from anti-ν)

MSW: mixing in matter

$$i \frac{d}{dt}\binom{\nu_e}{\nu_x} = H \binom{\nu_e}{\nu_x}$$

Via a rotation we can diagonalize H and get the diagonal H_m describing the evolution of the states which propagate as plane waves in matter ("matter eigenstates" ν_{1m} and ν_{2m} different from the mass eigenstates ν_1 and ν_2)

$$R_\theta = \begin{pmatrix} cos\theta_m & -sin\theta_m \\ sin\theta_m & cos\theta_m \end{pmatrix}$$

$$\binom{\nu_{1m}}{\nu_{2m}} = R_\theta \binom{\nu_e}{\nu_x}$$

$$H_m = R_\theta H R_\theta^{-1}$$

$$i \frac{d}{dt}\binom{\nu_{1m}}{\nu_{2m}} = H_m \binom{\nu_{1m}}{\nu_{2m}}$$

Best description in:
"Physics of Massive Neutrinos", F. Boehm and P.Vogel, Cambridge University Press.
Another good ref.: D. Perkins "Particle Atrophysics, Oxford Master Series in PPAC

MSW: Resonance condition

The newly defined mixing angle θ_m appearing in R_θ is related to the vacuum oscillation parameters and L_e

$$A = \frac{2\sqrt{2}G_F N_e E}{\Delta m^2} = \frac{L_V}{L_e}$$

$$L_V = \frac{4\pi E}{\Delta m^2}$$

$$L_e = \frac{4\pi}{2\sqrt{2}G_F N_e}$$

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - A)^2 + \sin^2 2\theta}$$

$$P(\nu_e \to \nu_x) = sin^2 2\theta_m sin^2 \frac{\pi L}{L_m}$$

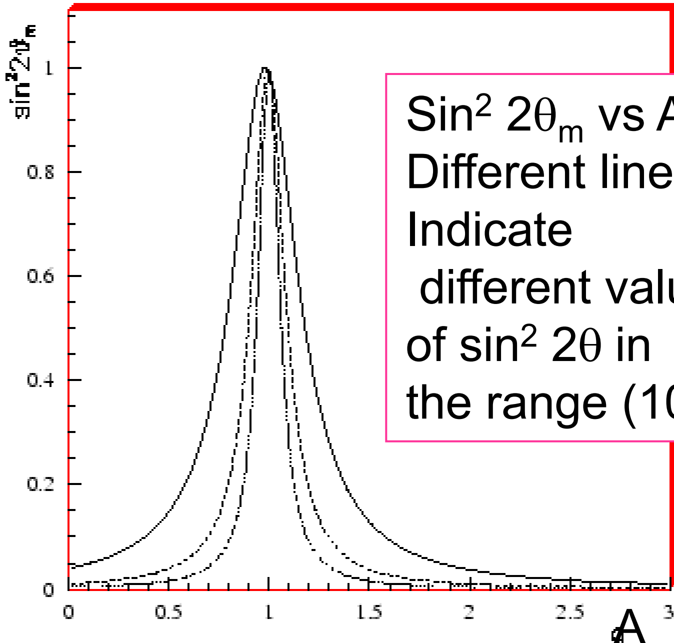
With $L_M = L_V \sin2\theta_m/\sin2\theta$

Resonance condition:

$L_v = L_e \cos2\theta$

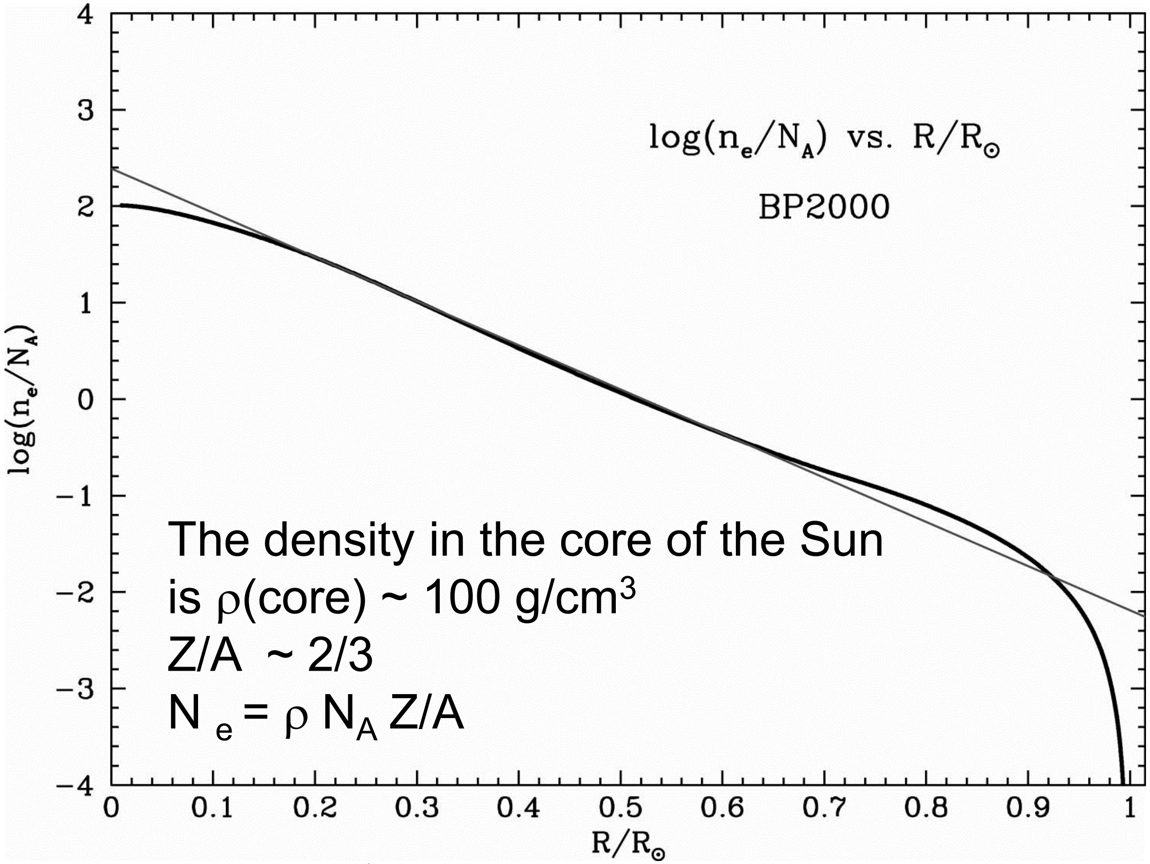
Maximal mixing in matter

Note that even if θ is very small at resonance electron-neutrinos could be transformed entirely in other active neutrinos via MSW



Sin² 2θm vs A
Different lines
Indicate
different values
of sin² 2θ in
the range (10^{-2}-10^{-3})

Matter effects in the Sun



The density in the core of the Sun is ρ(core) ~ 100 g/cm^3

Z/A ~ 2/3

$N_e = \rho N_A Z/A$

A rigorous treatment needs to take properly Into account the exponential fall of N_e

$L_e = 2\pi / (\sqrt{2}\, G_F N_e) \cong 1.7\ 10^7$ m /[ρ(g/cm^3) Z/A] $\approx$(Sun) 3×10^5 m

Solar radius = 3×10^8 m

Resonance condition is for $L_V = L_e \cos 2\theta$

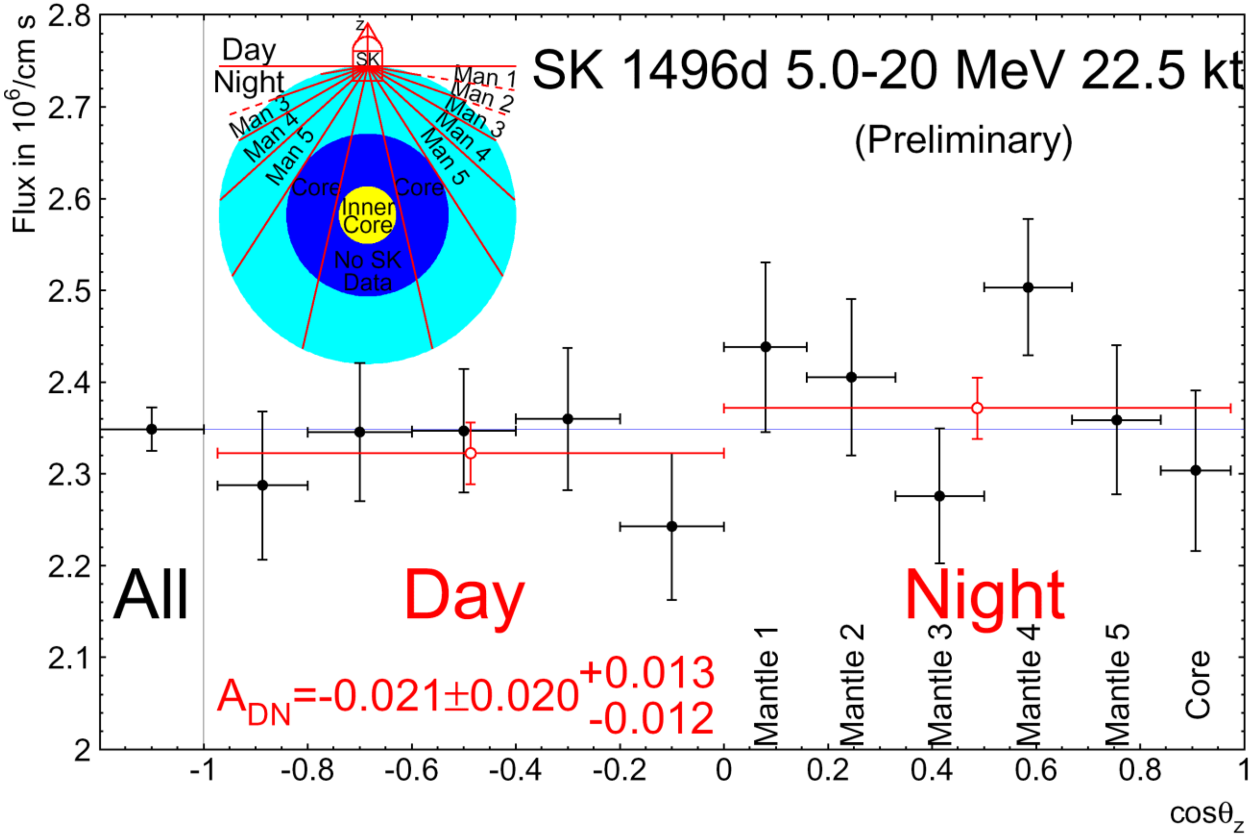
Take $\theta = 30^o$, resonance $\Rightarrow \Delta m^2 = 10^{-4}$–$10^{-5}$ eV^2

Matter effects in the Earth

The Number density of electron in the Earth is much less than in the SUN

$L_e = 2\pi / (\sqrt{2}\, G_F\, N_e) \cong 1.7\ 10^7 m / [\rho(g/cm^3)\ Z/A]$ ≈(rock ρ=3 g/cm^3 Z/A=1/2) 10^4 Km

Resonant MSW (for "LOW" Δm^2, 10^{-6}—10^{-7}) can produce a Day-Night asymmetry A_{DN} = N-D/N+D ***Sun may be brighter in the night!!***

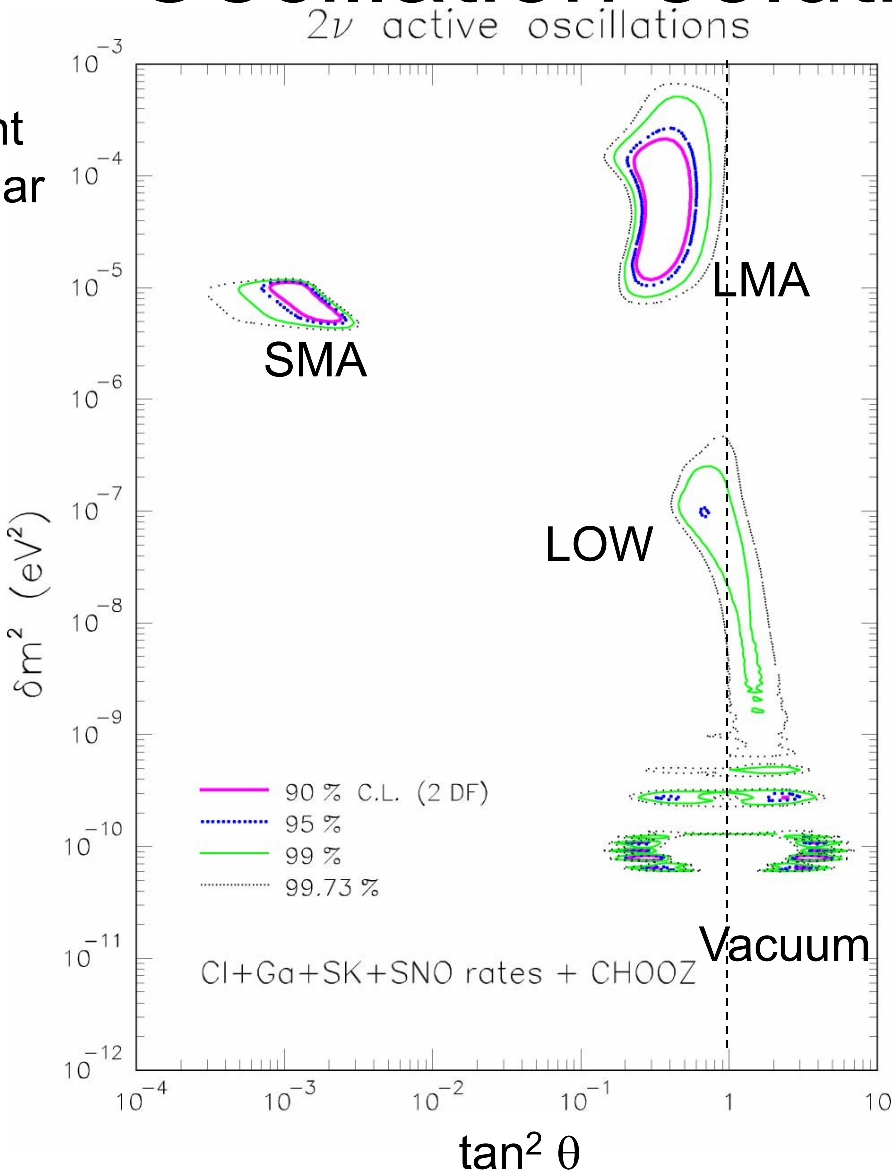


Until recently SuperK data were compatible with A_{DN} =0

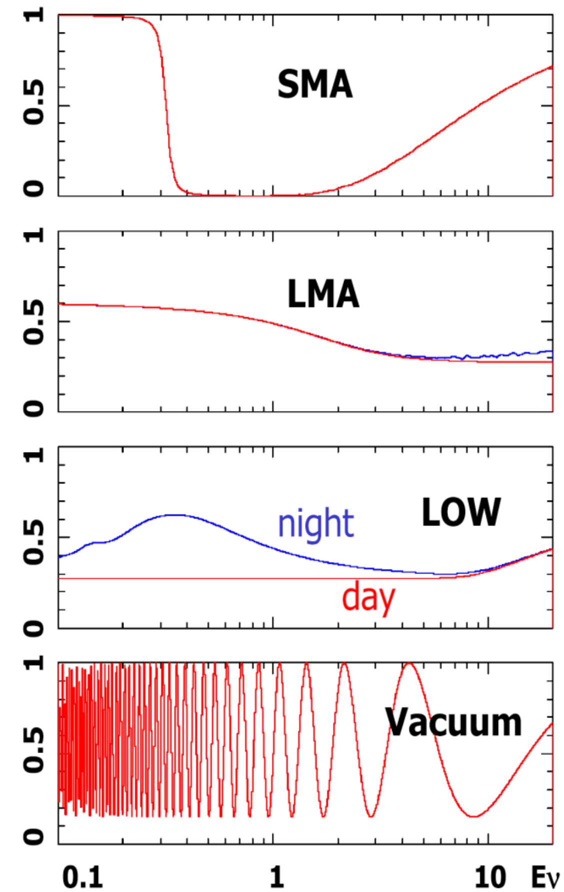
2015 update:
A_{DN} = -3.3 ±1.0± 0.5%
3σ non-zero significance

Oscillation solutions

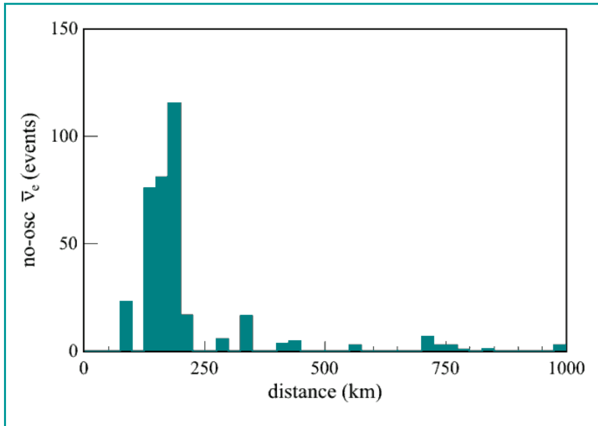
Fit to the event rates of all solar neutrino experiments (2002)

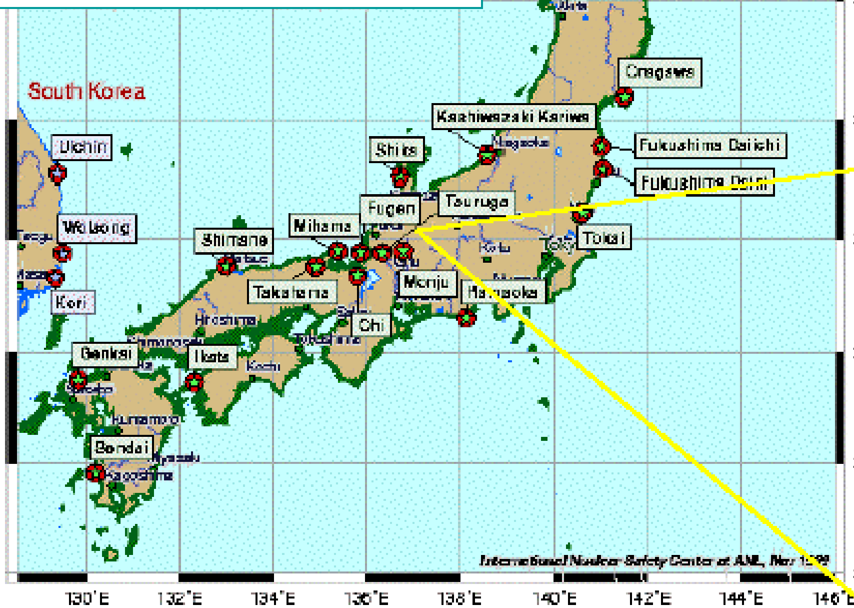


P(νe →νe)



From Sun to Earth: Kamland



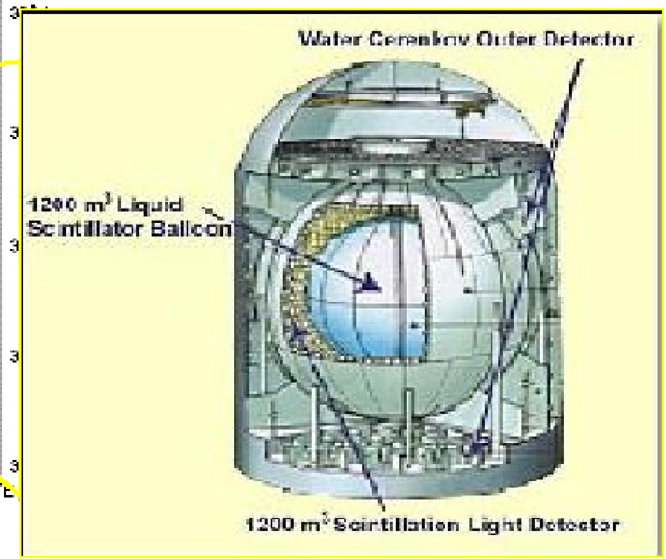


anti-ν_e from many powerful nuclear reactors in Japan (<L> ~ 200 Km) detected in 1Kton liquid scintillator

$$\overline{\nu_e} + p \rightarrow n + e^+$$

$$\downarrow$$

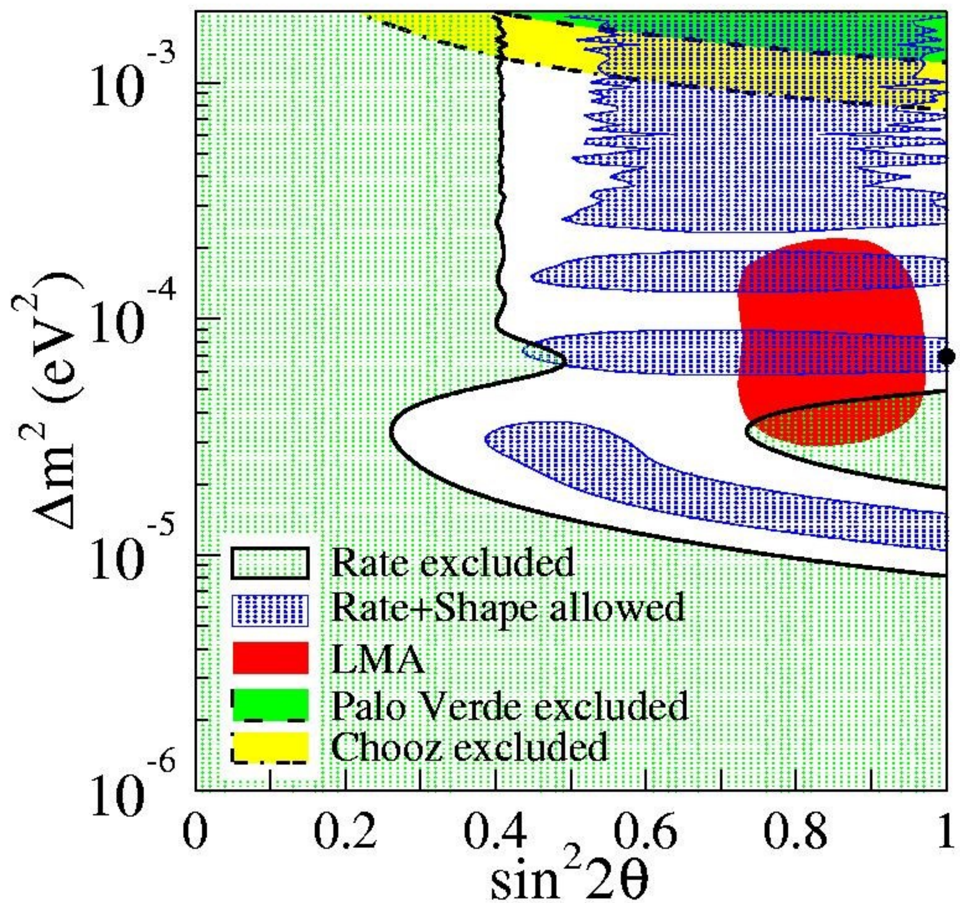
$$n + p \rightarrow d + \gamma$$

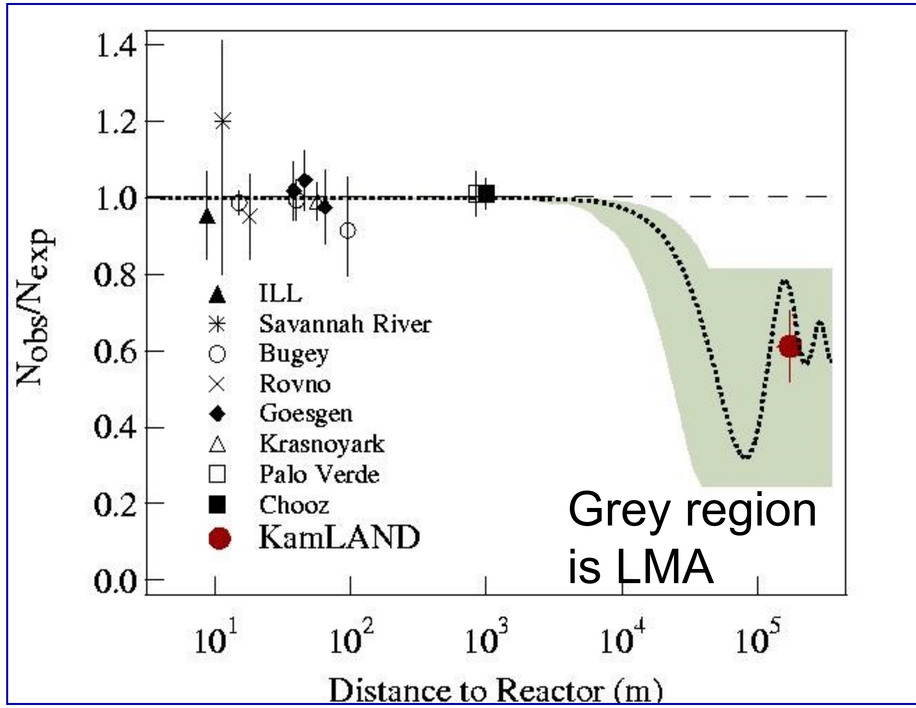


KamLAND confirms oscillations and selects LMA-MSW solution (12/2002)

Observed 54 events

Expected 86 ± 5.5

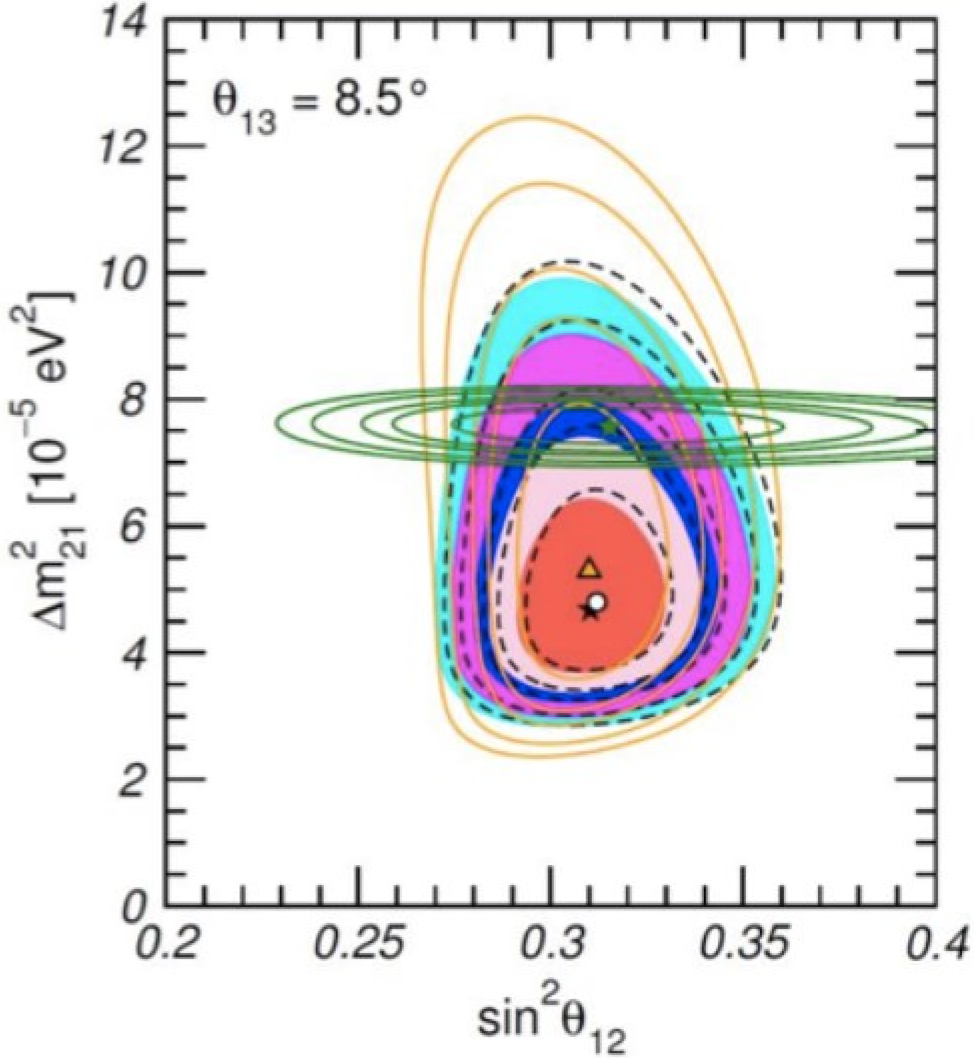




- Oscillation of reactor anti-ν_e proven
- Solar confirmed with man-made (anti)neutrinos

Allowed (Δm^2, $\sin^2\theta$) values by solar and Kamland results today

arxiv:1409.5439

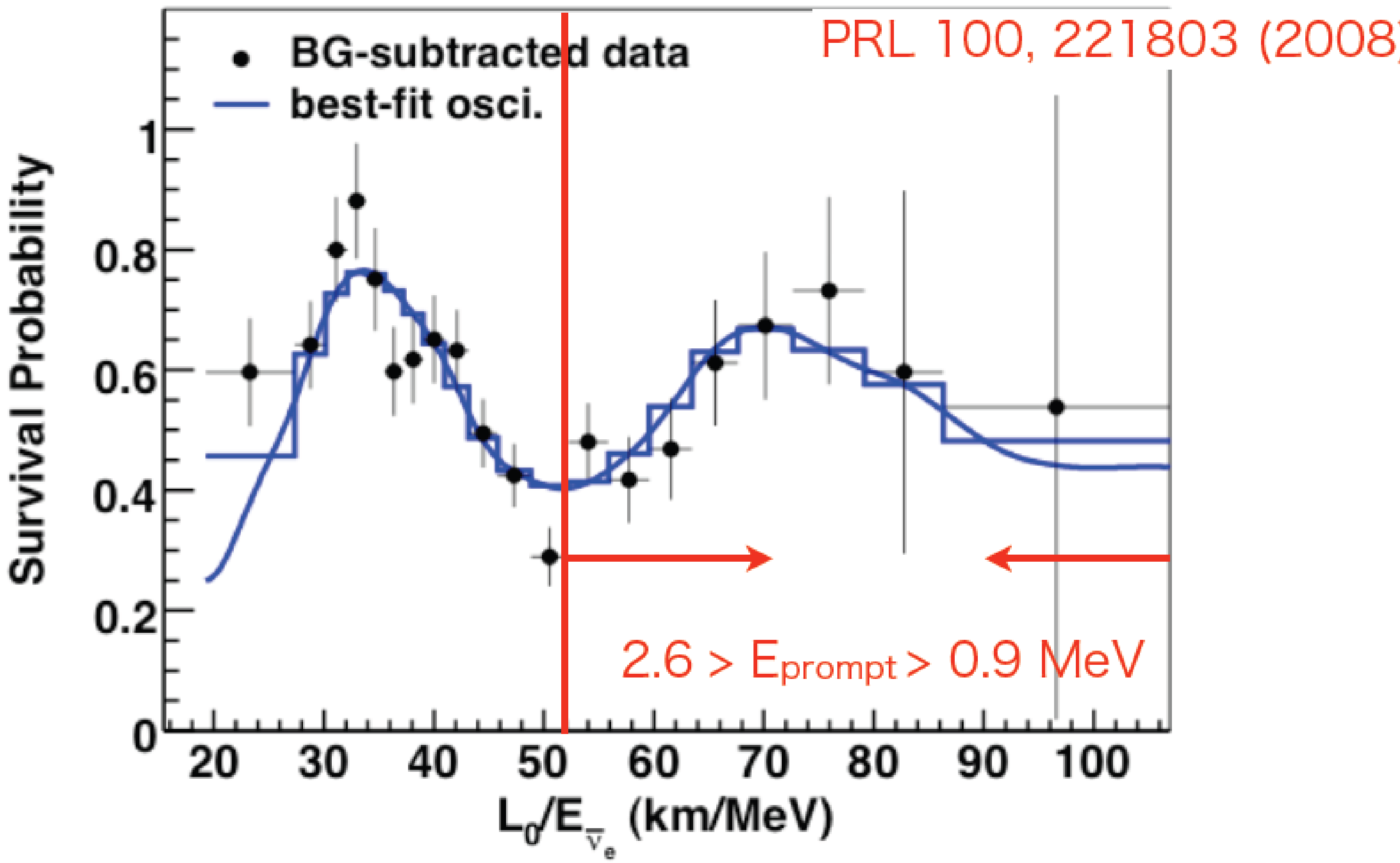


Allowed parameter regions (at 1, 90%, 2, 99% and 3 CL for 2 dof) from the combined analysis of solar data for GS98 model (full regions with best fit marked by black star) and AGSS09 model (dashed void contours with best fit marked by a white dot), and for the analysis of KamLAND data (solid green contours with best fit marked by a green star) for $\theta_{13} = 8.5°$.
Also shown as orange contours the results of a global analysis for the GS98 model but without including the day-night information from SK.

Kamland best Δm^2 precision

Kamland clearly sees the L/E dip!

$$P(\overline{\nu}_e \rightarrow \overline{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left(1.27 \Delta m^2 \frac{L_0}{E}\right) \qquad \langle L_0 \rangle = 180 \text{ km}$$



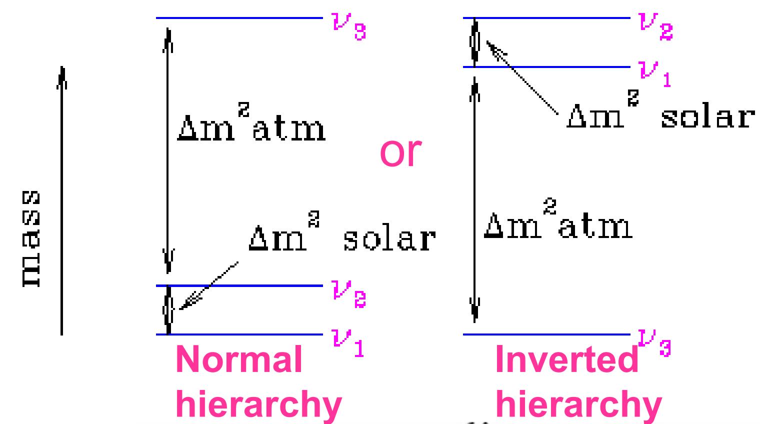
3-flavour ν oscillations

In general 3 ν weak-eigenstates are a superposition of 3 mass-eigenstates .

Actually we need 3-mass eigenstates to explain 2 different Δm^2 :

$|m_2^2 - m_1^2| = \Delta m^2_{sol} \sim 8\ 10^{-5}\ eV^2$

$|m_3^2 - m_2^2| = \Delta m^2_{atm} \sim 2\ 10^{-3}\ eV^2$

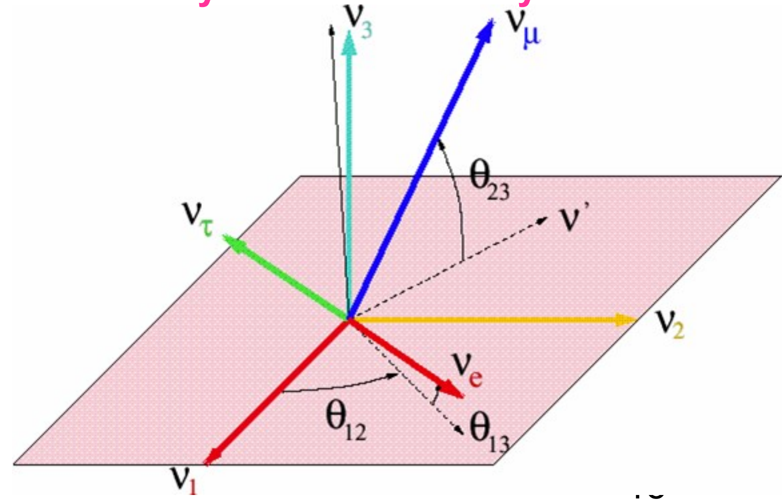


Neutrino mixing matrix or PMNS matrix

Relates mass and weak eigenstates

(analogue to CKM matrix in the quark sector)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



U_{PMNS} unitarity (as CKM again)

⇒ can be parameterized with 4 parameters: 3 angles, 1complex phase)

PMNS matrix

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Standard parameterization of Pontecorvo-Maki-Nakagawa-Sakata matrix

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{13} = \sin\theta_{13}$
$c_{13} = \cos\theta_{13}$

Atmospheric

SOLAR

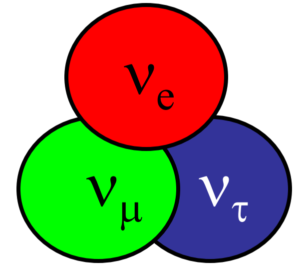
Solar & atmospheric ν oscillations easily accommodated within 3 generations.

Because of small $\sin^2 2\theta_{13}$, **solar & atmospheric ν oscillations almost decouple**

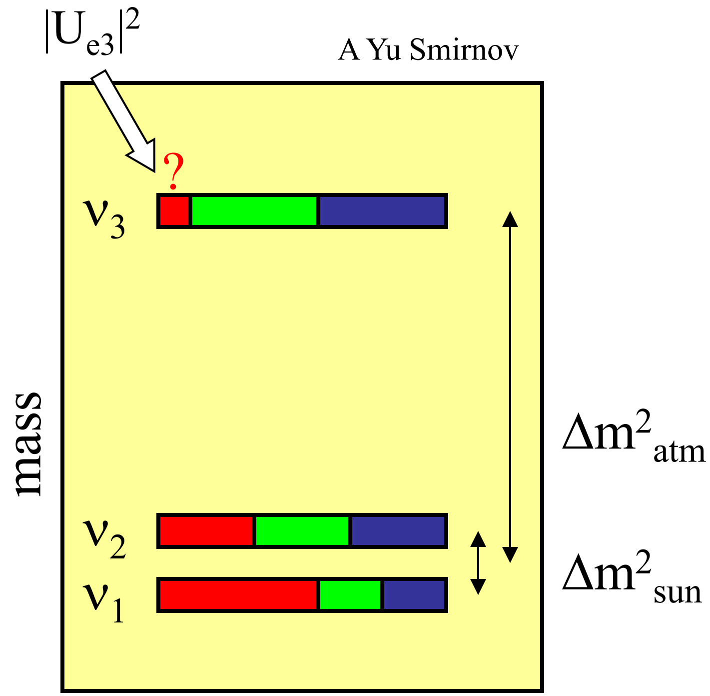
θ_{23} (atmospheric) ≅ 45^0
θ_{12} (solar) ≅ 33^0
θ_{13} (reactor) ≅ 8.5^0
δ?

$$U_{PMNS} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

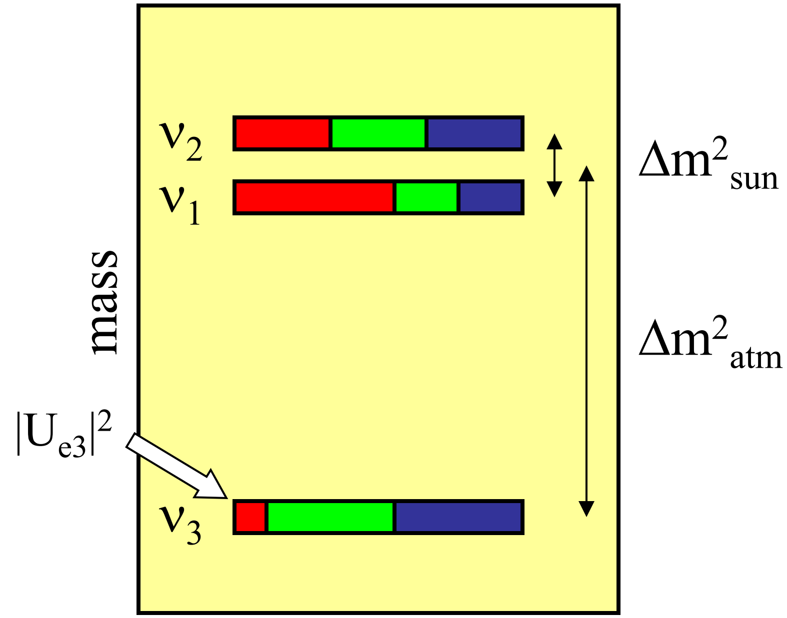
Mass spectrum and mixing



A Yu Smirnov



Normal mass hierarchy



Inverted mass hierarchy

We do not know yet:

- Absolute mass scale
- Type of the mass hierarchy: Normal, Inverted

Why does 2-flavour mixing work?

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$\left| \nu_\mu, t \right\rangle = |1\rangle U_{\mu 1}\, e^{-im_1^2 t/2p} + |2\rangle U_{\mu 2}\, e^{-im_2^2 t/2p} + |3\rangle U_{\mu 3}\, e^{-im_3^2 t/2p}$$

$$\left\langle \nu_\tau \right| = \langle 1| U_{\tau 1}^* + \langle 2| U_{\tau 2}^* + \langle 3| U_{\tau 3}^*$$

$$\left\langle \nu_\tau | \nu_\mu(t) \right\rangle = U_{\tau 1}^* U_{\mu 1}\, e^{-im_1^2 t/2p} + U_{\tau 2}^* U_{\mu 2}\, e^{-im_2^2 t/2p} + U_{\tau 3}^* U_{\mu 3}\, e^{-im_3^2 t/2p}$$

Why does 2-flavor mixing work?

If difference $|\Delta m_{12}|^2 << |\Delta m_{23}|^2$, we retrieve the 2-flavor formula: “one mass scale dominance”. This situation corresponds to an experiment whose L/E is such that the experiment can see only one mass splitting and is unable to resolve the 2 mass eigenstates in the other mass splitting, which are then seen as one. Like for atmospheric neutrinos.

$$\langle \nu_\tau | \nu_\mu(t) \rangle = U^*_{\tau 1} U_{\mu 1}\, e^{-i m_1^2 t / 2p}$$

$$+ U^*_{\tau 2} U_{\mu 2}\, e^{-i m_2^2 t / 2p} + U^*_{\tau 3} U_{\mu 3}\, e^{-i m_3^2 t / 2p}$$

$$\cong \left(U^*_{\tau 1} U_{\mu 1} + U^*_{\tau 2} U_{\mu 2} \right) e^{-i m_2^2 t / 2p} + U^*_{\tau 3} U_{\mu 3}\, e^{-i m_3^2 t / 2p}$$

$$= -U^*_{\tau 3} U_{\mu 3} e^{-i m_2^2 t / 2p} + U^*_{\tau 3} U_{\mu 3}\, e^{-i m_3^2 t / 2p}$$

$$\cong e^{i\delta} \sin\theta_{23} \cos\theta_{23} \left(-e^{-i m_2^2 t / 2p} + e^{-i m_3^2 t / 2p} \right)$$

It works also if one U coefficient is much smaller than the others. Since $U_{e3} \sim 0$, electron neutrinos couple to a good approximation only to 2 mass eigenstates, ν_1 and ν_2. Solar neutrinos case.

When is 3-flavor mixing important?

3-flavor mixing required in the interpretation of results by experiments sensitive to small values of θ_{13} (if $\theta_{13} \neq 0$!)

General formula for n flavors

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} + \\ - 4\sum_{i>j} \mathcal{R}eal(U^*_{\alpha i} U^*_{\beta i} U_{\alpha j} U_{\beta j}) \sin^2[(\Delta m^2_{ij} L)/(4E)] \\ + 2\sum_{i>j} Im(U^*_{\alpha i} U^*_{\beta i} U_{\alpha j} U_{\beta j}) \sin^2[(\Delta m^2_{ij} L)/(2E)]$$

We can then determine other sub-leading effects such as CP violation

Anti-neutrinos: the last term flips sign because $U \to U^*$

$\Rightarrow P(\nu_\alpha \to \nu_\beta) \neq P(\overline{\nu_\alpha} \to \overline{\nu_\beta})$

CP violation if 1 complex phase different from zero

$P(\nu_\mu \rightarrow \nu_e)$ on one slide (3 generations)

$$P(\nu_\mu \rightarrow \nu_e) = P_1 + P_2 + P_3 + P_4 + \text{corrections}$$

$$P_1 = \sin^2\theta_{23}\sin^2 2\theta_{13}\left(\frac{\Delta_{13}}{B_\pm}\right)^2 \sin^2\frac{B_\pm L}{2}$$

$$P_2 = \cos^2\theta_{23}\sin^2 2\theta_{12}\left(\frac{\Delta_{12}}{A}\right)^2 \sin^2\frac{AL}{2}$$

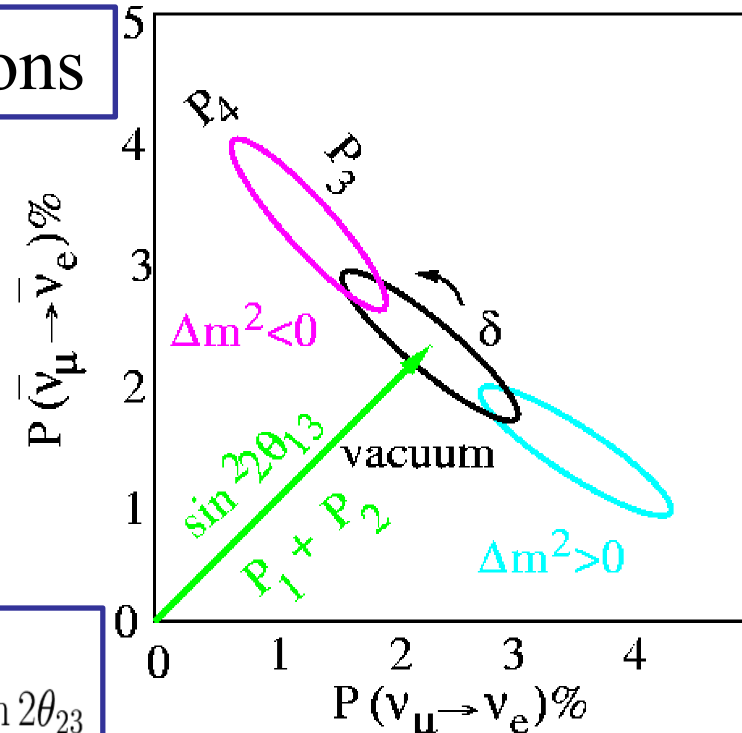
$$P_3 = J\cos\delta\left(\frac{\Delta_{12}}{A}\right)\left(\frac{\Delta_{13}}{B_\pm}\right)\cos\frac{\Delta_{13}L}{2}\sin\frac{AL}{2}\sin\frac{B_\pm L}{2}$$

$$P_4 = \mp J\sin\delta\left(\frac{\Delta_{12}}{A}\right)\left(\frac{\Delta_{13}}{B_\pm}\right)\sin\frac{\Delta_{13}L}{2}\sin\frac{AL}{2}\sin\frac{B_\pm L}{2}$$

$$\Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu} \qquad A = \sqrt{2}G_F n_e$$

$$B_\pm = |A \pm \Delta_{13}| \qquad J = \cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}$$

The ± is ν or $\bar{\nu}$



Minakata & Nunokawa JHEP 2001

Electron neutrino appearance in a muon-neutrino beam:

- Access to θ_{13}
- Access to δ_{CP} which enhances or suppress the conversion probability
- Matter effects also enhance or suppress probability
  - Matter effects depend on the mass hierarchy (sign of Δm^2_{13})
- In addition the probability probes the octant of θ_{23}